CHAPTER 10
VENN DIAGRAMS

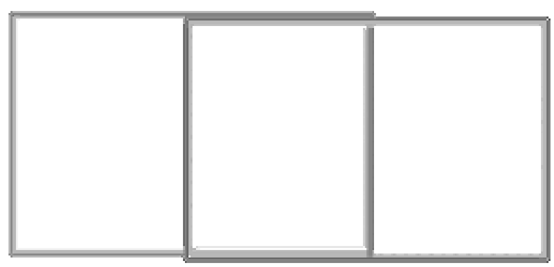
INTRODUCTION

In the nineteenth-century, John Venn developed a technique for determining whether a categorical syllogism is valid or invalid. Although the method he constructed relied on the modern interpretation of universal statements, we can easily modify it for use on the older classical view of such statements.

In the present chapter we will begin by explaining the technique as Venn originally developed it. Then later we will show you how to use it to determine validity on the older classical theory of categorical syllogisms.

REPRESENTING CATEGORICAL SYLLOGISMS WITH VENN DIAGRAMS

S *P*



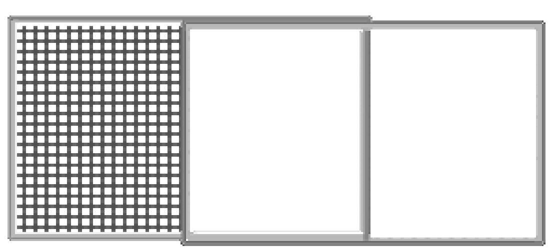
The two overlapping rectangles above should be construed as representing two sets. The one on the left represents the set consisting in all of the S things there are in the universe; while the one on the right represents all of the P things there are in the universe.

That part of the S rectangle which overlaps with the P rectangle represents those objects in the universe that are members of both sets. That portion of the S rectangle that does not overlap with the P rectangle represents all of the objects in the universe that are S but not P. While that portion of the P rectangle that does not overlap with the S rectangle represents all of the things in the universe that are P but not S.

SHADING

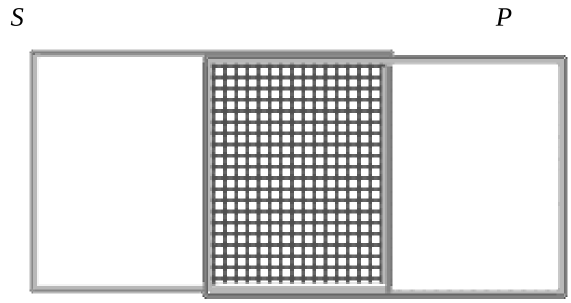
If we want to show that nothing is in a certain area we do this by shading that area. So, for example, if we want to say there are no Ss that are not Ps, we shade the area of the S rectangle that does not overlap the area of the P rectangle. We represent the claim that all S are P in this way.

S *P*



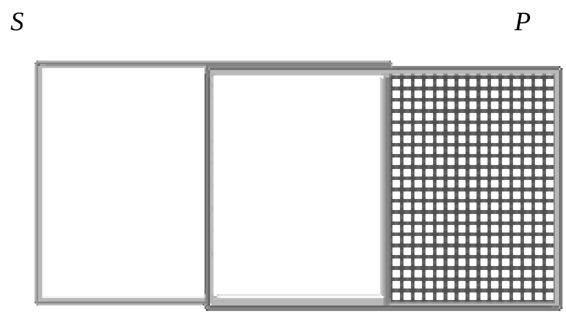
All S are P.

On the other hand, if we want to represent the claim that no Ss are Ps, we can do this by shading the area where the S rectangle and P rectangle overlap.



No S are P.

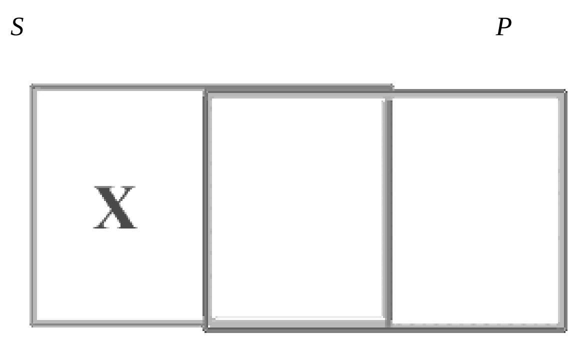
Finally, if we want to show that all P are S, or, in other words, there are no Ps that are not also Ss, we can shade the rightmost part of P.



All P are S.

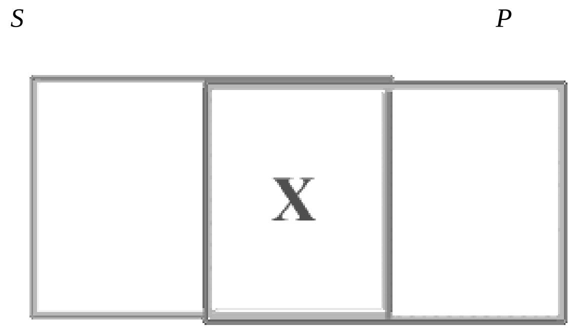
INDICATING THAT SOMETHING IS IN AN AREA

To show that something is in a certain area we place a capital X in that area. Thus, to say that there is something that is a member of S, but not a member of P, we place an X in the section of the S rectangle that does not overlap with the P rectangle.



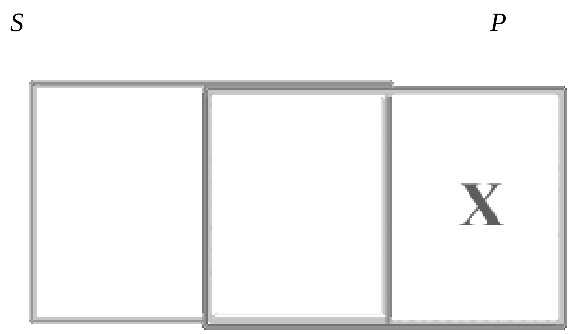
Some S are not P.

To say that something exists that belongs to both sets we place a capital X in the area where the two rectangles overlap.



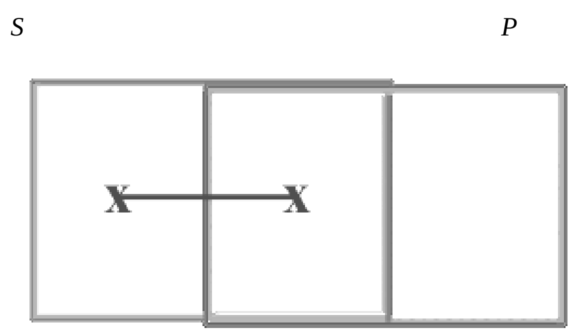
Some S are P.

To represent that something is a member of P, but not of S, we place an X in the area of the P rectangle that doesn't overlap the S rectangle.



Some P are not S.

Finally, if we know that the S set has something in it, but we don't know whether that thing is or is not also a member of P, we will place a lower case x in both sets and then connect them with a line. Thus, the diagram below tells us that there is at least one member of S, but it does not tell us whether that thing is, or is not, also a member of P.



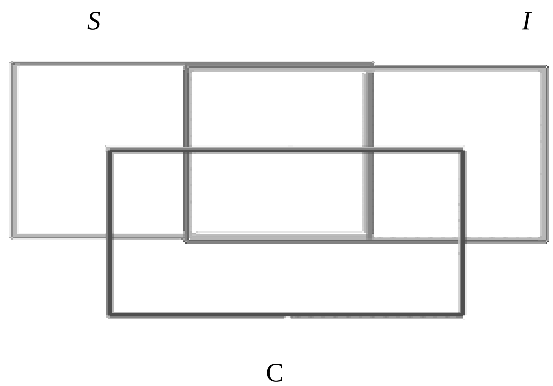
There are some S.

REPRESENTING CATEGORICAL SYLLOGISMS

So far we have been concerned with representing single statements in Venn diagrams. An argument, however, is not one statement, and a categorical syllogism is a type of argument. More specifically, a categorical syllogism is an argument that contains exactly two premises, both of which are categorical statements. The problem is how do we use Venn diagrams to represent categorical syllogisms? And how do we use them to decide whether such syllogisms are valid?

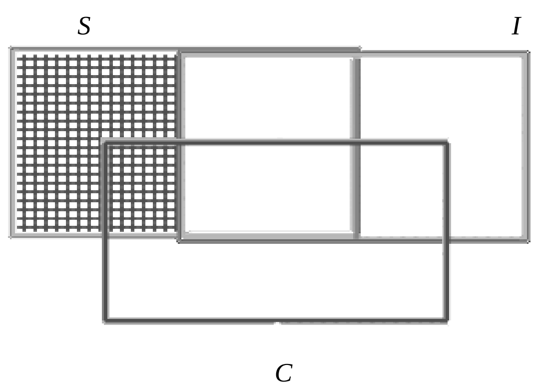
Perhaps the first thing to notice is that a categorical syllogism refers to three sets, rather than two. So, instead of two overlapping rectangles we will need three such rectangles.

EXAMPLE 1



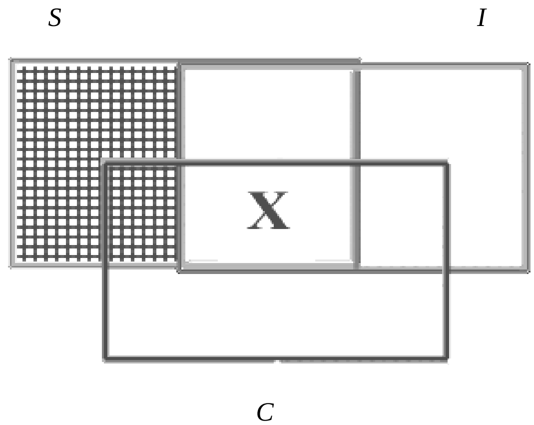
All schools are educational institutions.
Some schools are colleges.
____
Some colleges are educational institutions.

To decide whether the argument above is valid or invalid we must begin by representing both of the premises in the diagram. We represent the first premise by shading the area of schools that are not educational institutions.



All schools are educational institutions.
Some schools are colleges.
____
Some colleges are educational institutions.

Then we need to represent the second premise. This premise tells us that there is at least one school that is also a college. To express this in the diagram, we need to place an x in those areas where schools and colleges overlap. The only such area that has not been shaded, however, is the area where the three rectangles overlap. So we know that something exists in this area. We must, therefore, place a capital X in this area and our diagram will then look like this:

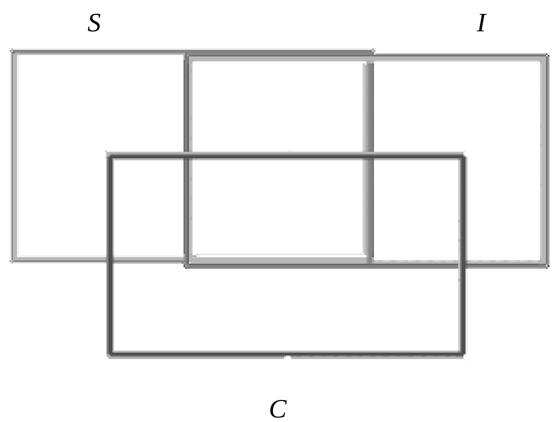


All schools are educational institutions.
Some schools are colleges.
____
Some colleges are educational institutions.

All that remains is to evaluate whether the conclusion of the argument follows from its premises. If it does the argument is valid; otherwise it is invalid. Clearly the argument in question is valid.

Let's try one more example.

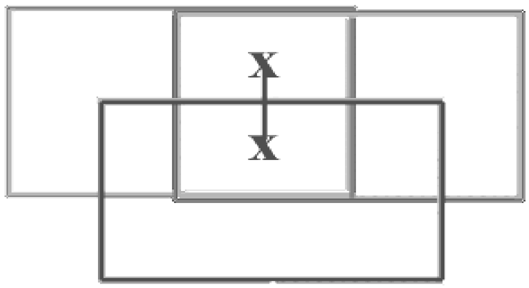
EXAMPLE 2



Some schools are educational institutions.
Some schools are colleges.
____
Some colleges are educational institutions.

To represent the first premise of this argument, we need to show that something is in the area of schools that are educational institutions. In the diagram there are, however, two areas that represent schools that are educational institutions and, unfortunately, the first premise doesn't tell us whether the schools that are educational institutions are, or are not, colleges. So we need to place a lowercase x in both areas and draw a line between them.

S I



C

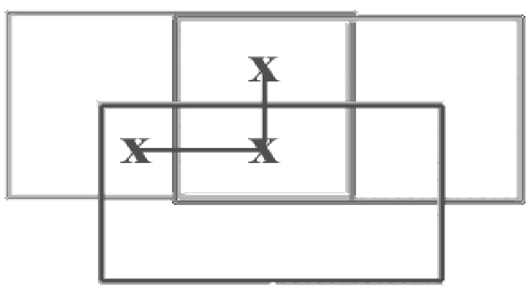
Some schools are educational institutions.
Some schools are colleges.

____

Some colleges are educational institutions.

Clearly we have to represent the second premise in a similar way, except that we need to x-x the areas where schools and colleges overlap. Once we have done this our diagram will look like this:

S I



C

Some schools are educational institutions.
Some schools are colleges.

____

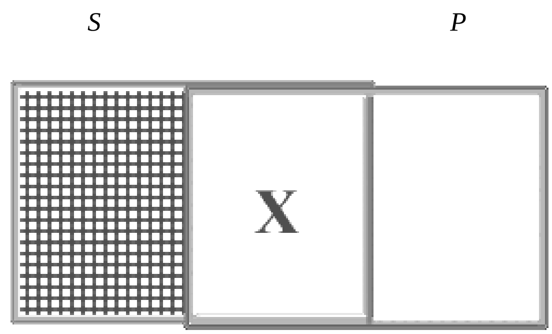
Some colleges are educational institutions.

Evidently the conclusion of this argument does not follow from its premises since the conclusion informs us that something is definitely in the area where colleges and educational institutions overlap but the diagram doesn't show this.

USING VENN DIAGRAMS TO DETERMINE VALIDITY ON THE CLASSICAL VIEW

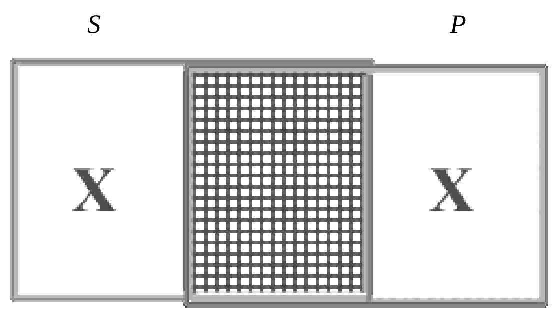
To use Venn diagrams to determine validity on the classical view we need to alter the above account. Classical logic evidently assumes that in sentences which express universal categorical statements, both the subject and predicate terms refer to existing objects. So on the classical view, the statement that all S are P entails that there exists an S that is a P. While the statement that no S are P entails not only that there exists an S that is not a P, but also that there exists a P that is not an S.

If we want to use Venn diagrams to determine validity on the classical view we need to add these assumptions to the diagram. So if the statement is an A-statement, besides shading the area of S that is not P, we must also add an X to the area where S and P overlap. Doing this, we obtain. . . .



All S are P.

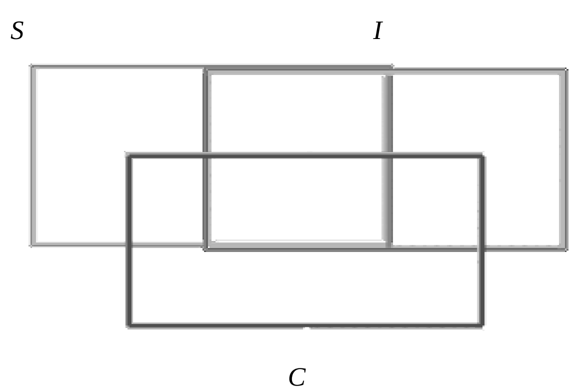
On the other hand, if we want to represent the statement that no S are P, besides shading the area where the S and P rectangles overlap, we also need to place an X in the area of the S rectangle that is not in the P rectangle, and in the area of the P rectangle that is not in the S rectangle. Doing this we get:



No S are P.

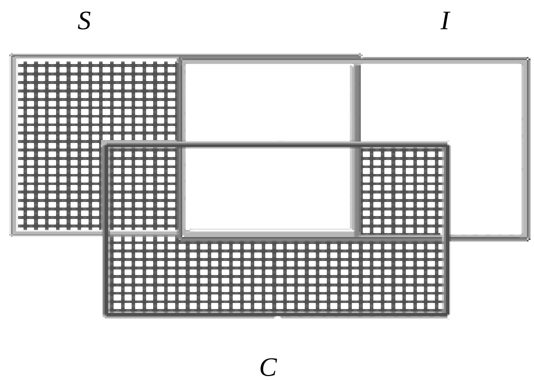
Let's see how this will work with some actual syllogisms.

EXAMPLE 1



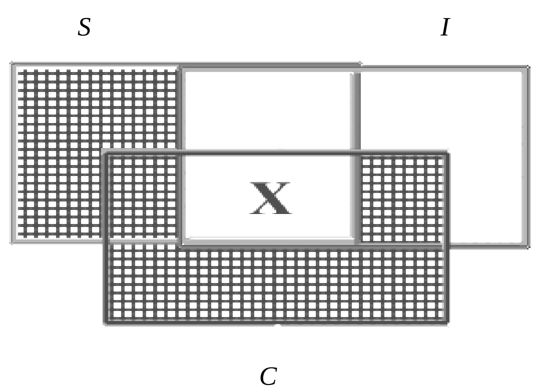
All colleges are schools.
All schools are educational institutions.
____
Some colleges are educational institutions.

The easiest way to handle this is to begin by representing the premises in the same way we would if we were adopting the modern perspective. Accordingly, we shade all of those areas of schools that are not educational institutions and of colleges that are not schools. Thus, we will shade the diagram as suggested below. (Notice that if this were the end of the matter we would have to evaluate the argument as invalid, since the conclusion does not follow from the premises.)



All colleges are schools.
All schools are educational institutions.
____
Some colleges are educational institutions.

However, we are not finished yet. We must now add the assumptions that the premises make about the sets referred to in their subject terms. Here this means that we must place a capital X in the area that represents colleges that are schools, since this is implied by the first premise.



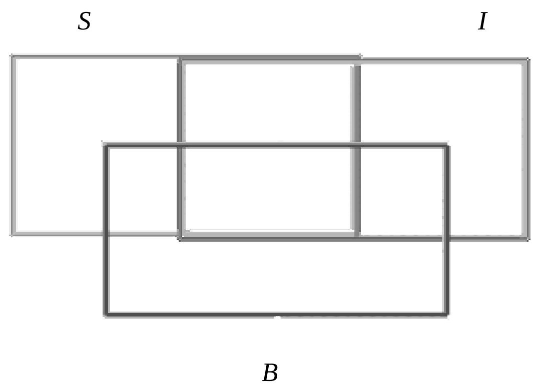
All colleges are schools.
All schools are educational institutions.
____
Some colleges are educational institutions.

Normally, we would also have to place lowercase x-x in the two areas where schools and educational institutions overlap, since this is an assumption that the second premise commits us to. Here, however, since the earlier premise already commits us not only to the existence of some colleges, but also to existence of some schools, this is unnecessary.

Clearly, our diagram shows that the argument is valid on the classical view.

NOTE: Any argument that is valid on the modern view will also be valid on the classical view; and any argument that is invalid on the classical view will also be invalid on the modern view. (The converses of these principles are, however, not true.)

EXAMPLE 2

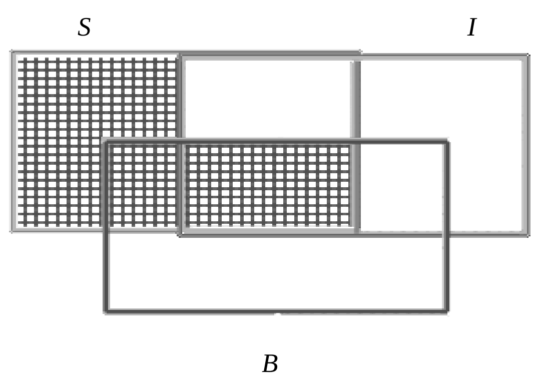


All schools are educational institutions.
No schools are bars.

____

Some bars are not educational institutions.

As we suggested in the last example, we begin by representing the premises just as we would if we were adopting the modern view of syllogisms. Doing this, we get. . . .

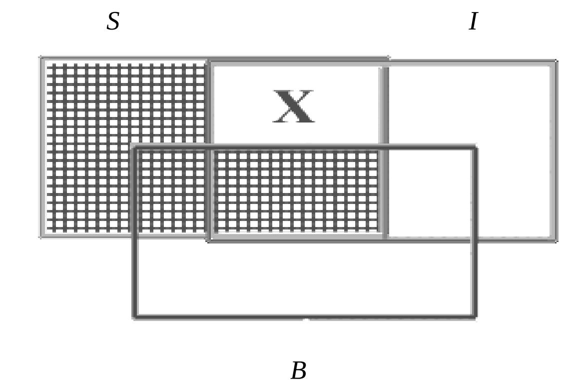


All schools are educational institutions.
No schools are bars.

____

Some bars are not educational institutions.

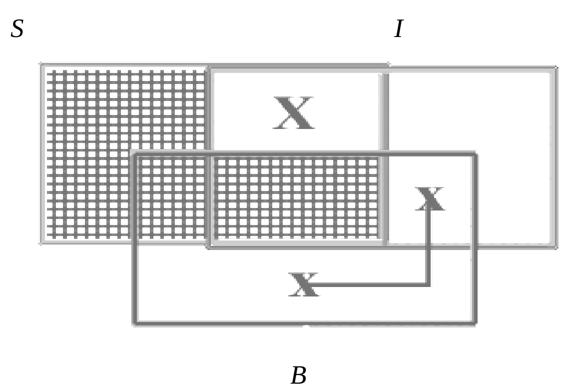
Now, however, we need to add the additional assumptions that the classical view makes about the premises. Since the first premise commits us to the existence of at least one school that is an educational institution, we need to represent this by placing a capital X in the un-shaded area where schools and educational institutions overlap.



All schools are educational institutions.
No schools are bars.

Some bars are not educational institutions.

To represent the presuppositions that classical logic makes about the second premise we must also add in our diagram that there exist some schools that are not bars and some bars that are not schools. However, we have already represented the first of these assumptions in the diagram. So all we need to do then is to represent the assumption that some bars exist that are not schools. We do this by x-ing the area of bars that are educational institutions and the area of bars that are not educational institutions, and drawing a line between them.



All schools are educational institutions.
No schools are bars.

Some bars are not educational institutions.

Inspecting the diagram we have now completed, we see that the conclusion of the argument does not follow from its premises. It is, therefore, invalid.

A WORD OF WARNING

All of this may seem clear enough, and so long as the things we are reasoning about exist, the principles of classical logic are acceptable. When we begin reasoning about objects that don't exist, however, problems arise. First, the principle that the contradictory of any A statement must have the opposite truth-value of that statement does not work. Thus, suppose for example, the statement is that all unicorns are beautiful animals. This statement is false on the classical view, because there are no unicorns; while the contradictory, "Some unicorns are not beautiful animals," is also false for precisely the same reason. Moreover, both the I-statement "Some unicorns are beautiful animals," and its contradictory "No unicorns are beautiful animals," will be false for the same reason. Second, the principle that subcontraries cannot both be false must also be abandoned. For both the I-statement that some unicorns are beautiful animals, and the O-statement that some unicorns are not beautiful animals, are false.

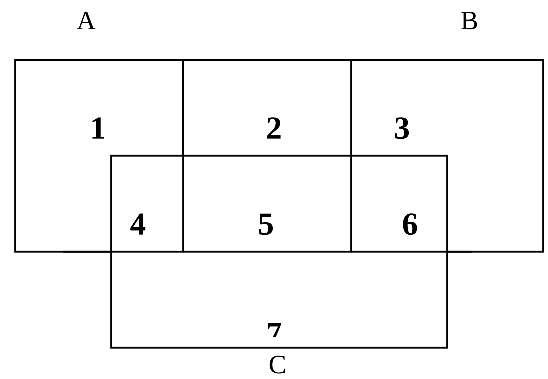
Whenever we are dealing with arguments that refer to nonexistent objects, we must represent them from the modern perspective. Unfortunately, this is a severe limitation of classical logic. Many logicians believe that logic should be independent of the way the world actually is. Whether an argument is valid or invalid should not depend on whether or not its terms refer to existing objects.

SOME IMPORTANT POINTS TO REMEMBER

1. Whenever a premise is a universal statement you should always shade; and whenever it is an existential statement you should always x.
2. With any kind of premise, always shade or x in the rectangle that represents the statement's subject term.
    - a.) If the premise reads, "All S are P," always shade the areas of the S rectangle that are outside the P rectangle.
    - b.) If the premise reads, "No S are P," always shade those areas where the S and P rectangles overlap.
    - c.) If the premise reads, "Some S are P," find the two areas of the S rectangle that overlap the P rectangle. If neither of these areas is shaded, place an x-x between them. If one of the two areas is shaded, place a capital X in the other area.
    - d.) If the premise reads, "Some S are not P," find the two areas of the S rectangle that do not overlap with the P rectangle. If neither of these areas is shaded, place an x-x between them. If one of the two areas is shaded, place a capital X in the other area.
3. If you are working within the classical interpretation, after proceeding as indicated above, if either of the premises is a universal statement:
    - a.) If the premise reads, "All S are P," you must indicate that there is at least one thing in the areas where the S and P rectangles overlap. If one of the two areas where these rectangles overlap is shaded, place a capital X in the other area. If neither of these areas is shaded, place x-x between them.
    - b.) If the premise reads, "No S are P," you must show that there is at least one thing in the area of the S rectangle that does not overlap the P rectangle, and that there is at least one thing in the area of the P rectangle that does not overlap the S rectangle. If either of the two areas where the S rectangle doesn't overlap the P rectangle is already shaded, you should place a capital X in the other area. If neither of these areas is shaded, place x-x between them. Then do the same for the area of the P rectangle, which does not overlap the S rectangle.
4. When an area is shaded, it means nothing is in that area. When a capital X is in an area, it means that something is definitely in that area. When an x-x exists between two areas, it means something exits in one of the two areas. An empty area tells you nothing.
5. An argument is valid if the diagram forces you to admit that its conclusion is true. Otherwise, it is invalid.

HELP

If you have not understood the discussion above then proceed as follows: Number the three rectangles as indicated below. Then identify the four numbers that constitute the rectangle of the statement's subject. Write them down. Underneath these four numbers write the four numbers that constitute the rectangle of the statement's predicate. Then use the chart below to either shade or x-x two areas.



	The two common numbers between the subject and the predicate	The two numbers that are different in the top set of numbers
Shade	E	A
x-x	I	O

HOW TO HANDLE AN A-STATEMENT

Suppose the claim reads “All B are C.” The four numbers that represent the subject’s rectangle (i.e., the B rectangle, since B occupies the subject position in the sentence) are 2356. The four numbers that represent the predicate’s rectangle (i.e., the C rectangle, since that occupies the predicate position in the sentence) are 4567. Since the statement is an A-statement the chart above tells us to find and shade the two numbers that are different in the subject’s rectangle. Those numbers are 2 and 3. So the chart above tells us to shade 2 and 3. (Read across and up from the A in the chart above.)

If we are representing things from the classical perspective we have additional work to do. We need to represent Aristotle’s claim that a true A-statement implies a corresponding true I-statement in the diagram. So after we have proceeded as indicated in the paragraph above the chart above tells us to x-x the two common numbers between the subject and predicate. Those numbers are 5 and 6.

HOW TO HANDLE AN E-STATEMENT

Suppose the claim reads “No A are C.” The four numbers that represent the subject’s rectangle (i.e., the A rectangle, since A occupies the subject position in the sentence) are 1245. The four numbers that represent the predicate’s rectangle (i.e., the C rectangle, since that occupies the predicate position in the sentence) are 4567. Since the statement is an E-statement the chart above tells us to find and shade the two numbers that are common between these two sets of numbers. Those numbers are 4 and 5. So the chart above tells us to shade 4 and 5. (Read across and up from E in the chart above.)

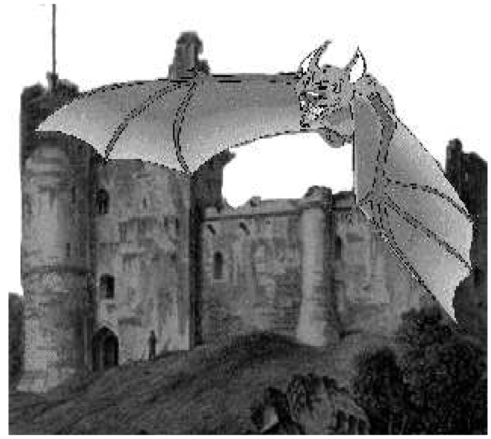
If we are representing things from the classical perspective, here again we have additional work to do. We need to represent Aristotle’s claim that a true E-statement implies a corresponding true O-statement in the diagram. The chart above tells us to x-x the two numbers that are different in the top set of numbers. These numbers are 1 and 2.

But on Aristotle’s view the converse of a true E-statement has the same truth value as the original. So the claim that “No A are C” implies that “No C are A,” and this in turn commits us to the O-statement “Some C are not A.” To represent this claim in the diagram we must find and x-x the two numbers in the bottom set that are different from the numbers in the top set. Those numbers are 6 and 7. So we should also x-x these two areas.

HOW TO HANDLE I-STATEMENTS AND O-STATEMENTS

If the claim reads “Some C are B” the diagram above tells us we need to x-x areas 5 and 6; while if it reads “Some C are not B” we need to x-x areas 4 and 7. No extra work is required if we are constructing the diagram on the Classical perspective.

EXERCISES



Instructions: Construct two Venn Diagrams on the syllogism below, one of which represents that argument on the modern view, and the other of which represents it on the classical perspective. Decide whether the argument is valid or invalid on each view.

All vampires are bats.
No vampires are hemophiliacs.
____
Some hemophiliacs are not bats.

Instructions: Construct two Venn Diagrams on each of the arguments below, one of which represents that argument from the modern view, and the other of which represents it on the classical view. Determine whether the argument is valid or invalid on each of the two perspectives.

1. No diamonds are opals. No diamonds are sapphires. So no sapphires are opals.
2. Some islands are vacation resorts. All islands are paradises. So some paradises are vacation resorts.
3. All teachers are alcoholics. No alcoholics are politicians. So some politicians are not teachers.
4. No pleasurable experiences are headaches. All IRS audits are headaches. So no IRS audits are pleasurable experiences.
5. All dragons are fire hazards. No endangered species are fire hazards. So some endangered species are not dragons.
6. Only funny people are clowns. Funny people are never tedious oafs. So, all tedious oafs are non-clowns.

A BRAINTEASER

Instructions: Construct Venn-like Diagrams on the argument below and determine whether it is valid or invalid on both the classical and modern interpretations.

Since all vampires are bats and some vampires are bloodsuckers, but no hemophiliacs are bats, it follows that some bloodsuckers are not hemophiliacs.

LIMITATIONS AND A PROLOGUE

Although they are effective in determining the validity of many arguments containing quantifiers, both the syllogistic and diagrammatic approaches we have been exploring in the last two chapters are somewhat limited, most especially because they are unable to effectively evaluate arguments with premises involving two or more quantifiers. They cannot, for example, determine the validity of the argument that since everyone loves a lover and someone loves someone, it follows that everyone loves everyone. How these sorts of arguments are to be dealt with was not discovered until the invention of quantification theory in the early part of the twentieth century.