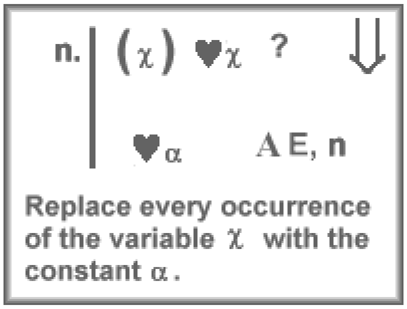
CHAPTER 12B
QUANTIFICATION THEORY
PROOFS: OTHER

This system expands on the one developed in Chapter 7B. All the rules used there are applicable here, together with the following new ones.

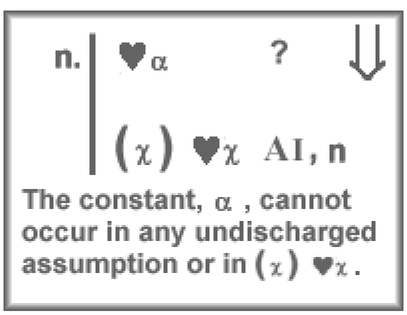
UNIVERSAL QUANTIFIER ELIMINATION



The rule Universal Quantifier Elimination tells us how to use a universally quantified formula. We should view the rule as a two-step process. It tells us first to drop the quantifier, and then select a constant--a lower-case letter from a to u--and replace every remaining occurrence of the variable that appeared in that quantifier with that constant. Study the example below.

1	(x)(Px>Qx)		Premise
2	Pa	/ Qa	Premise
3	(Pa>Qa)		AE, 1
4	Qa		>E, 2, 3

UNIVERSAL QUANTIFIER INTRODUCTION



Universal Quantifier Introduction tells us how to create a universally quantified formula. It tells us that we can select a formula, a constant, and a variable, and replace every occurrence of that constant in the formula with that variable. We then precede this with a left parenthesis plus that variable plus a right parenthesis. Unfortunately, there is an important restriction on using this rule. The constant we choose cannot occur in any un-discharged assumption or in any premise. Examine the examples below carefully.

AN EXAMPLE OF A LEGAL USE OF THE RULE AI

1	(x)Px		Premise
2	(y)Qy	/ (x)(Px.Qx)	Premise
3	Pa		AE, 1
4	Qa		AE, 2
5	(Pa.Qa)		. I, 3, 4
6	(x)(Px. Qx)		AI, 5

AN EXAMPLE OF A LEGAL USE OF THE RULE AI

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6	(x)(Px. Qx)		AI, 5

EXAMPLES OF ILLEGAL USES OF THE RULE AI

1) Violates the Restriction

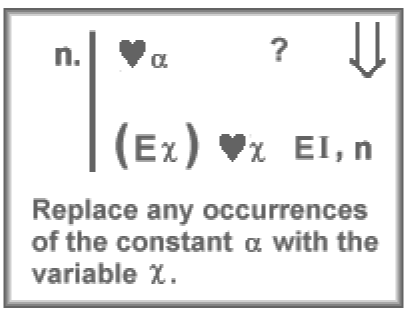
1.	(x)Pax	Premise
2.	Paa	AE, 1
3.	(x)Pxx	AI, 2

← *NO* →

2) Doesn't Replace All Occurrences of 'a'

1.	(x)Pxx	Premise
2.	Paa	AE, 1
3.	(x)Pax	AI, 2

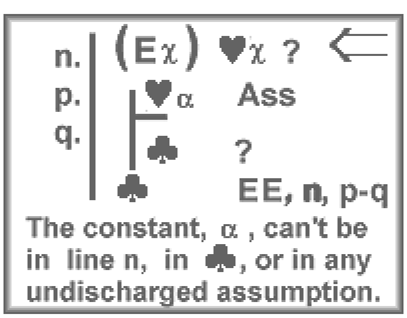
EXISTENTIAL QUANTIFIER INTRODUCTION



Existential Quantifier Introduction allows us to create a formula whose main operator is an existential quantifier. The rule is very easy. To use it all we need to do is select a variable, a constant, and a formula. We then replace any number of occurrences of the constant we have chosen with that variable in that formula. We then precede the formula with a left parenthesis, then the existential quantifier *E*, followed by the variable we chose, and then a right parenthesis.

1	((Ex)(Ey)Pxyy>Qbb)		Premise
2	Paaa	/ (Ex)Qbb	Premise
3	(Ey)Payy		EI, 2
4	(Ex)(Ey)Pxyy		EI, 3
5	Qbb		>E, 1, 4
6	(Ex)Qbb		EI, 5

EXISTENTIAL QUANTIFIER ELIMINATION



Of all the rules in this system, Existential rule is the most complex. To use the rule we must have an existentially quantified formula already listed. We then need to assume an instance of this formula. (To obtain an instance of the formula, chop off the quantifier, and replace every occurrence of the variable that occurred in that quantifier, with a constant.) Under this assumption we then need to derive a formula. The rule tells us we can stop the assumption we made. Moreover, we can move the formula we derived under it to the left of that assumption, if we have not violated the following restrictions:

1. The constant we selected when we assumed an instance of the existentially quantified formula cannot occur in the formula we are moving over.

2. This constant also cannot occur in any un-discharged assumption.

3. The constant cannot occur in the original existentially quantified formula.

LEGAL

1	(Ex)Pxx	Premise
2	Paa	Assume
3	(Ey)Pyy	EI, 2
4	(Ey)Pyy	EE, 1, 2-3

ILLEGAL

1	(Ex)Pxa	Premise
2	Paa	Assume
3	(Ey)Pyy	EI, 2
4	(Ey)Pyy	EE, 1, 2-3

ILLEGAL

1	(Ex)Pxx	Premise
2	Paa	Assume
3	(Ey)Pay	EI, 2
4	(Ey)Pay	EE, 1, 2-3

EXERCISES