CHAPTER 4
TRANSLATION

SIMPLE AND COMPOUND STATEMENTS

Let's begin with some distinctions that should, hopefully, help you learn to translate effectively. Then we will show you how to translate.

We will classify statements into two groups. Some statements are compound and others are simple. A statement is compound if it is possible to view it as made up of another statement, or other statements. A statement is simple if it not possible to view it as made up of any other statements.

The statement, "Shakespeare wrote Romeo and Juliet," is a simple statement. We cannot view it as made up of any other statements. On the other hand, the statement, "Either Milton or Shakespeare wrote Romeo and Juliet," is compound. For we can view this statement as made up of the statements, "Milton wrote Romeo and Juliet," and "Shakespeare wrote Romeo and Juliet."

We can further subdivide compound statements into two types. Some compound statements are truth-functionally compound, while others are non-truth-functionally compound. A statement is truth-functionally compound if the truth or falsity of its component statements is sufficient to determine whether it is true or false. Otherwise, it is non-truth-functionally compound.

To see the difference between these, compare the following two statements:

1. It is not the case that Milton wrote Romeo and Juliet.
2. Martha thinks that Milton wrote Romeo and Juliet.

Both these statements are compound because both contain the simple statement, "Milton wrote Romeo and Juliet." While Statement 1 is truth-functionally compound, however, Statement 2 is non-truth-functionally compound. Statement 1 is truth-functionally compound because, to know that it is a true claim, all we need to know is that the component "Milton wrote Romeo and Juliet," is a false claim. Statement 2, on the other hand, is non-truth-functionally compound. For even if we do know that the statement "Milton wrote Romeo and Juliet," is false, this will not suffice to tell us whether the statement, "Martha thinks that Milton wrote Romeo and Juliet" is true or false. To know this we would need to know what Martha thinks -- and it's doubtful that even Martha knows that.

Let's try one more example. Compare the two following statements:

1. Although Shakespeare wrote Romeo and Juliet, Milton wrote Paradise Lost.
2. Milton wrote Paradise Lost after Shakespeare wrote Romeo and Juliet.

Each of these two statements is compound, but only the first one is truth-functionally compound. Statement 1 is truth-functionally compound, because if both of the component claims, "Shakespeare wrote Romeo and Juliet," and "Milton wrote Paradise Lost," are true, then it will be true. On the other hand, if either of the component claims is false, then statement 1 will be false.

In contrast, Statement 2 is non-truth-functionally compound. This is so because the fact that both component claims are true does not decide the truth or falsity of Statement 2. To figure out the truth or falsity of Statement 2, we also need to know which work was written later.

As you will see, we are going to be focusing on those statements that are truth-functionally compound, and we will be treating them differently from both the simple and the non-truth-functionally compound statements. In fact, for purposes of translation it won't much matter whether a statement is simple or non-truth-functionally compound. So let's group both of these into one category and call them "atomic statements" and we will call all and only truth-functionally compound statements "molecular statements."

Besides containing at least one component statement, every compound statement contains a word or an expression that is not itself a statement, but hooks onto statements to build more complex statements. Such expressions are called "connectives." Connectives come in two varieties. Some are unary connectives, while others are binary connectives. A unary connective is a word or expression that hooks onto a single statement to form another statement. On the other hand, a binary connective connects two statements together to form a third statement.

In the examples we have been considering, the phrases "It is not the case that" and "Martha thinks that" function as unary connectives. When we attach them to the left of a statement, we get a larger statement. In our first example, we attached the connective, "It is not the case that" to the left of the atomic statement, "Milton wrote Romeo and Juliet," to form the molecular statement, "It is not the case that Milton wrote Romeo and Juliet." Obviously, we could just as easily have attached it to the left of any statement to form another statement.

In our later examples we used the binary connectives, "although" and "after" to glue two statements together. Thus, we used "although" to connect "Shakespeare wrote Romeo and Juliet," and "Milton wrote Paradise Lost," to form the statement, "Although Shakespeare wrote Romeo and Juliet, Milton wrote Paradise Lost." While we used the connective "after" to construct "Milton wrote Paradise Lost after Shakespeare wrote Romeo and Juliet" from these two simple statements.

QUESTIONS

Consider the statement:

In spite of the fact that Fred failed because he didn't study enough, he did learn a lot from his Ancient Greek History class.

1. Is it simple or compound?
2. Is the statement truth-functionally compound or is it non-truth-functionally compound?
3. How many connectives are contained in it?
4. How many simple statements are contained in it?
5. Is the statement, "Fred failed because he didn't study enough," truth-functionally compound?
6. Is the statement "He (Fred) didn't study enough," simple or compound?
7. Is the statement "He (Fred) didn't study enough," truth-functionally compound?

THE SYMBOLS

If you were going to teach someone how to play chess you might begin by picking each piece out of the box, holding it up, and labeling it. Although the person you were teaching the game to would not know how these pieces functioned, at least he or she would have an idea of what the game looked like. You might then begin to teach him what counted as a legal move in the game.

Before we actually begin learning to translate, we want to do something very much like this. We want to name the various symbols we will be using in the game we will be playing, and tell you what counts as a legal move in that game. Although this may not be terribly exciting, and we suspect you want us to begin teaching you to translate as quickly as possible, we are convinced that this material is extremely important.

One type of symbol we will be using in our game of logic is called "an atomic statement letter." An atomic statement letter is any capitalized letter in the alphabet. The letter "A," is, therefore, an atomic statement letter, and so is "B."

A second type of symbol we will be using is called "a connective." In the system we will be working with there are five different connectives. Only one of these is an unary connective. The other four are all binary connectives.

The only unary connective we will be using is called "a tilde" (pronounced "tilda"). In most books it is represented as ~, but we prefer to use the negative sign, -, instead.

One of the binary connectives we will be using is called "a dot." While some logicians have recently begun using an ampersand, &, to represent this connective, we will use . to represent it, since this is both easy to type and write.

Another of the four binary connectives we will be using is called "a wedge." This connective is virtually always represented v, and we will also be representing it in this manner.

We call the third of our four binary connectives "a horseshoe." In many logic texts this connective is represented as a backward c, though recently, some logicians have begun using a right arrow. We have decided, however, to use the greater than symbol, >, because it is not only familiar but also both easy to type and write.

Our fourth binary connective has often been represented as a triple bar or a double arrow. We will, however, use an equal sign, =, to represent this connective.

Besides atomic statement letters and connectives, our symbolic language also contains punctuation marks. Unlike English, however, which uses a variety of punctuation marks (e.g., commas, semicolons, colons, and periods), our symbolic language uses only two symbols of punctuation. One of these is a left parenthesis, (, while the other is a right parenthesis,).

The only other symbols we will be using can be referred to as special symbols. These are the left and right curly braces, viz., { and }, a backslash, and a semicolon.

ATOMIC STATEMENT LETTERS	CONNEC-TIVES	PUNCTUA-TION	SPECIAL SYMBOLS
A B C . . . Z	- . v > =	( )	{ } / ;

WELL-FORMED FORMULAE

In English, certain groupings of symbols are meaningful, while others are not. Thus, "Milton wrote after Shakespeare died," makes sense, while "Wrote Milton Shakespeare after died," does not. The same is true of our symbolic language. No doubt the rules for constructing a meaningful grouping of symbols in English are incredibly complicated, but fortunately this is not true of our symbolic language. In fact, only four rules are required to explain what constitutes a meaningful grouping of symbols (called a well-formed formula):

1. Any atomic statement letter is a well-formed formula.
2. If ♣ is a well-formed formula, so is: -♣
3. If ♥ and ♣ are well-formed formulae, so are:

(a) (♥.♣)
(b) (♥v♣)
(c) (♥>♣)
(d) (♥=♣)

Note that, unlike the tilde, which never causes parentheses, one pair of parentheses surrounds each of these connectives.

4. No other groupings of symbols are well-formed formulae.

Let's see how these rules work.

Rule 1 tells us, for example, A, and C, are both well-formed formulae. Moreover, since A is a well-formed formula, Rule 2 tells us that -A is also a well-formed formula. However, if we apply Rule 2 again, but this time to the well-formed formula -A, we get the result that --A is also a well-formed formula. We have now discovered that --A, and C, are both well-formed formulae. Rule 3 (b) implies that, since this is so, (--AvC) is also a well-formed formula. However, because this is so, and since -A is a well-formed formula, Rule 3(c) informs us that ((--AvC)>-A) is also a well-formed formula. Finally, Rule 2 establishes that, since this is so, -((--AvC)>-A), is also a well-formed formula.

Rule 4 simply tells us that no other strings of symbols are well-formed formulae. So it tells us, for example, ((A.B.C)>-)D is meaningless.

Before continuing, let's briefly review what we have learned about the symbolic language.

QUESTIONS

Consider the following concatenation of symbols: -(-A=-(B.-(Cv--D)))

1. Is this a well-formed formula?
2. How many tildes occur in the formula?

3. How many binary connectives occur in it?

Consider the following collection of symbols: -(-(-G)v-H)

4. Is it a well-formed formula?

Now consider the following group of symbols: (L-M)

5. Is it a well-formed formula?

What about this group of symbols? (B.C.E)

6. Is it a well-formed formula?

TRANSLATION: "NOT," "AND," AND "OR"

We are finally ready to begin learning how to translate from English into our symbolic language. The translation process always begins with a translation key. In this key, each atomic statement in the passage we want to translate is assigned one atomic statement letter. Thus, suppose for example, we want to translate the following passage into symbols:

> Neither Bob nor Joe will go camping unless Diane goes too. But Diane won't go if either Bob or Helen goes. So Joe won't go camping.

The translation key for this passage would look like this:

Translation Key:

B: Bob will go camping.
D: Diane will go camping.
H: Helen will go camping.
J: Joe will go camping.

Once we have set up our translation key, all we need to do is to express the claims made in symbols. If the statement made is atomic it is expressed by simply writing down the appropriate atomic statement letter. So if we want to express the claim that Bob will go camping, we simply write B.

On the other hand, if the claim we want to express is molecular, not only will it contain at least one connective in English, its symbolic representation must also contain at least one of our symbolic connectives.

Suppose the claim we want to translate into symbols is, "Bob won't go camping." As we saw earlier, this claim can be rewritten as, "It is not the case that Bob will go camping." When it is rewritten in this way we can see not only that it is truth-functionally compound, but that it contains the atomic statement "Bob will go camping," plus the unary connective, "It is not the case that."

Now unlike the atomic statement letters, which vary in meaning from context to context, our five connectives always mean the same thing. As you may already have guessed, our only unary connective, the tilde, is always used to mean "It is not the case that." Since this is so, to represent "Bob won't go camping," all we need to do is write -B.

In ordinary English, double negatives are frowned on. But our symbolism permits them. Even ----B is permitted. It might be translated, "It is not true that it isn't the case that Bob won't not go camping." (Quite a mouthful, isn’t it?)

The primary meaning of the dot is "and." So "Bob and Helen will go camping," is represented as (B.H). Notice that, unlike the tilde, but like all of the other connectives, the dot always has a formula on both its left and right sides, and is always surrounded by a pair of parentheses.

Besides "and," the dot is also used to represent a host of other connectives in English. Of these connectives the following are among the more common ones: "but," "however," "although," "while," "moreover," "too," "also," "in addition to," and "as well as." The dot represents the idea of conjunction. We use it whenever we want to say this plus that.

Suppose we wanted to say that Bob will go camping, but Helen won't. We would represent this in symbols as (B.-H). While the claim, "Bob won't go camping, but Helen will," would be represented: (-B.H). Compare this claim with the claim, "It is not true that both Bob and Helen will go camping," and you will see why the parentheses are important. This latter claim is not only different in English from the claim that "Bob won't go camping, but Helen will," it is also different in symbols. For it is represented: -(B.H).

Suppose we wanted to represent the statement, "Bob, Joe, and Helen will all go camping." We might be tempted to express this in symbols as (B.J.H). Oddly enough, this is wrong. The mistake is a technical one, but it's very important. Since the formula contains two binary connectives, it must also have two left and two right parentheses. Unfortunately, (B.J.H), isn't a well-formed formula. We must represent the claim as either: ((B.J).H), or (B.(J.H)).

Our next binary connective is the wedge. The primary meaning of this connective is "or." Unfortunately, however, in English there are two different senses of the word "or," and the conditions under which they are true vary slightly. In one of these senses, called "the exclusive sense of 'or'," the word "or" has the force of "either, or, but not both." When we go to a restaurant and see "soup or salad" on the menu, this is the sense of "or" that is being used. There is, however, also an inclusive sense of this word. Unlike the exclusive sense, the inclusive sense of "or" means to allow the possibility of both disjuncts being true. It has the force of "and/or." When it states in an insurance policy that the company pays in the event of death or disability, it is this sense that is being used.

We cannot use the same connective to represent both of these senses of "or," since, unlike "and" and "but," for example, they vary in the conditions under which they are true. In every logic text we are aware of the wedge is always used to mean the inclusive sense of "or" only, and that is also the way we are going to use it. So "v" means "and/or." (We will see how to represent the exclusive sense of "or" shortly.)

In everyday life it is not always entirely clear when someone says "or," which of the two senses is intended, and this could create translating nightmares. For our purposes, however, this really won't be a problem at all. Whenever logicians say "or" you should assume that they are using the inclusive sense (unless, of course, they explicitly say, "or, but not both."), and just translate it as a wedge.

The only other major expression in English that is translated as a wedge is "unless." This confuses many people because they think "unless" and "or" have quite different meanings. In fact, however, "unless" and "or" are quite similar. If this translation of "unless" puzzles you, for the present you might just try to remember that "unless" is a wedge. Later, once we have developed the symbolic system more, we will be able to justify this interpretation.

Before going on to the last two connectives let's practice some translations using the three connectives we have discussed.

QUESTIONS

For this set of exercises we will provide the translation key. We will also give you the statement to be translated. You represent the statement in symbols.

Translation Key:

B: The butler committed the crime.
G: The gardener committed the crime.
M: The maid committed the crime.
S: The secretary was asleep.

1. Both the butler and the gardener committed the crime.
2. The butler and the gardener committed the crime while the secretary was asleep.
3. In spite of the fact that the secretary wasn't asleep, the maid committed the crime.
4. Either the butler or the maid committed the crime.
5. Neither the butler nor the maid committed the crime.
6. Either the butler committed the crime, or both the maid and the gardener did it.
7. The crime was committed by either the butler or the maid, but not both.
8. Either the crime was committed by both the butler and the gardener or both the butler and the maid.
9. It isn't true that unless the secretary wasn't asleep both the maid and butler committed the crime.

TRANSLATION: "IF," AND "IF AND ONLY IF"

We'll try some more exercises translating shortly. First, however, let's learn how to use our two remaining connectives.

Of all the connectives, the horseshoe is the most difficult. It is used to represent a conditional statement. A conditional statement is a statement that sets down a condition, and then goes on to talk about what is the case if that condition is met. The condition is frequently referred to as "the antecedent," while the part of the statement that goes on to say what is true, given that the antecedent is met, is called "the consequent."

With respect to the horseshoe, the claim that sets the condition down (i.e., the antecedent) is always put on its left side, while the consequent is always placed on its right side. This is extremely important.

The primary meaning of the horseshoe is, "If . . . then . . . " Thus the statement, "If Shakespeare wrote Romeo and Juliet, then Milton wrote Paradise Lost," will be represented as (S>M). Note that the antecedent of this conditional statement is that Shakespeare wrote Romeo and Juliet, and it, therefore, belongs on the left side of the horseshoe. The symbolic claim, (M>S) would say, "If Milton wrote Paradise Lost, then Shakespeare wrote Romeo and Juliet."

Unfortunately, all sorts of complications arise with the horseshoe. First, the word "then" may not occur in the statement. For it might read, "If Shakespeare wrote Romeo and Juliet, Milton wrote Paradise Lost." Insofar as the symbolic translation is concerned, this doesn't matter. The claim is still translated: (S>M). Second, the consequent might be expressed in the sentence before the antecedent. Thus, someone might say, "Milton wrote Paradise lost, if Shakespeare wrote Romeo and Juliet." This is still translated as (S>M), because the antecedent is still that Shakespeare wrote Romeo and Juliet.

To make matters even worse, there are many other words besides "if" that set a condition down, words like: "since," "because," "when," "for," and "provided that," to mention at least a few. (You should start keeping a list of these words.)

A third difficulty arises with two specialized cases. Many people have troubles with the expression, "only if." While they recognize that this connective in English needs to be translated as a horseshoe (since it sets down a condition) they choose the wrong side as the antecedent. They represent "Shakespeare wrote Romeo and Juliet only if Milton wrote Paradise Lost," as (M>S). While this would be a correct translation of "Shakespeare wrote Romeo and Juliet, if Milton wrote Paradise Lost," it is not correct for "only if." The claim should be represented: (S>M). (An easy way for you to remember how to represent "only if" is just to remember you should place the horseshoe where the connective "only if" occurs.)

For some reason, the expression, "if and only if," may also cause you problems. Often enough, people try to represent it as a horseshoe. Thus, they translate "Shakespeare wrote Romeo and Juliet if and only if Milton wrote Paradise Lost," as (S>M). Actually, this is not a strong enough claim. "If and only if" expresses a condition going in both directions. If anything, it should be translated as: ((M>S).(S>M)). For we can see it as saying, "Shakespeare wrote Romeo and Juliet if Milton wrote Paradise Lost, and Shakespeare wrote Romeo and Juliet only if Milton wrote Paradise Lost." As you will see shortly, however, there is a much simpler way of treating "if and only if."

One further problem with translating conditional statements might be worth at least briefly mentioning. There are, in fact, a number of different kinds of conditional statements in English; not all of which are translated with a horseshoe, because not all of them express truth-functionally-compound statements. Frequently the expression, "if . . . then . . . ." has the force of, "if this then afterwards that." Unfortunately, the system of logic we are constructing is not powerful enough to express connectives that suggest a temporal sequence. To make matters worse, there is a use of expressions like "if . . . then . . ." that suggests a causal connection (e.g., "If the bridge collapsed, then there was too much weight on it.") This kind of causal conditional is also too sophisticated for the system we are building. Technically, it should be treated as atomic and translated by assigning an atomic statement letter to it.

Fortunately, from the point of view of this text, or any translation exercises you might work on in other logic texts, our discussion of the different kinds of conditional statements can be pretty well ignored. When the author of the exercises uses "if . . . then . . . " you will probably be on safe grounds if you just translate the claim as a horseshoe.

The last of our five connectives is, perhaps, the easiest one of all. The = represents the idea of a condition going in both directions. It is rarely used in English. When it is used, however, it is almost always expressed either as "if and only if," or as "just in case." When you see these expressions, just use =. Otherwise, it’s probably a horseshoe you want.

There are just two further points we want to make before we start practicing. The first concerns punctuation marks in the English sentence. You should pay very careful attention to any commas or semicolons that occur. They indicate major breaks in the sentence, and they are frequently immediately followed by a connective. Moreover, a semicolon is much more significant than a comma. If the word immediately following the semicolon is a connective, then the symbolic connective you use to represent it should have fewer parentheses around it than any other connective in the formula.

The last point may not be one you want to hear. It is this: Translation is something of an art. It requires practice. As you practice translating into symbols more, you will become better at it. When you make a mistake try to see what you did wrong, and try to remember not to make that mistake again. Above all, be patient.

Without more ado, let's try some more exercises. These involve the use of all five of our connectives.

QUESTIONS

As before, we will provide the translation key, and you translate each of the claims below into symbolic notation.

Translation Key:

B: The butler committed the crime.
G: The gardener committed the crime.
M: The maid committed the crime.
S: The secretary was asleep.

1. If the secretary was asleep, then both the butler and gardener committed the crime.
2. If both the maid and gardener committed the crime, then the secretary was asleep.
3. The maid committed the crime only if the gardener didn't.
4. The maid committed the crime if the gardener didn't.
5. The butler committed the crime if and only if the gardener didn't.
6. It is not true that if the butler committed the crime then the gardener didn't.
7. If not both the butler and gardener committed the crime, then the maid did it while the secretary was asleep.
8. Although, the butler didn't commit the crime if the maid did, he did commit it if the gardener did.
9. If neither the maid nor the gardener did it, then the butler did it while the secretary was asleep.
10. It isn't true that the maid did it provided that both the butler and the gardener did it.
11. If the secretary was asleep, then the maid didn't commit the crime if the butler did it.
12. It's false that if the secretary wasn't asleep the maid committed the crime only if the butler didn't.

SETS OF STATEMENTS AND ARGUMENTS

So far our only concern has been with translating single statements. But we also need to learn to translate sets of statements and arguments. Before concluding this chapter, perhaps we should at least briefly discuss these.

Some time ago, when we were discussing the various symbols in the symbolic language, we mentioned several symbols, which were identified as special symbols. These included the left and right curly braces, { and }, the backslash, /, and the semicolon, ;. These special symbols are used when we are representing sets of statements and arguments.

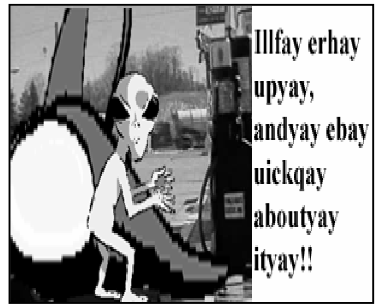
A set of statements is, as you may recall, simply a collection of claims that we have decided to view as a unit. We need a way of representing these which will distinguish them from both single statements and arguments.

We have decided to represent a set of statements by simply surrounding the statements in the set with a pair of curly braces and separating the statements with semicolons. So, for example, suppose the set we are interested in contains the statements: (P>-Q), (R=--S), and -(-TvG). We will express this in symbols as: {(P>-Q);(R=--S);-(-TvG)}.

We will represent an argument in a similar manner. We will glue the premises together with semicolons and surround them with a set of curly braces, just as we did with a set of statements. We will then use a backslash, and write the conclusion immediately after the backslash. Thus, for example, we will represent it as: {(P>(Q.R));(Q>-T);T}/-P. That's all there is to it.

PROBLEMS

Instructions: Using the translation key provided, translate the argument below into symbolic notation.



Translation Key:

A: The attendant has a coronary.
F: The anti-gravity device fails.
G: Alien Bob gets gas.
M: Alien Bob has his MasterCard.
W: The attack on Washington will succeed.

Alien Bob gets gas assuming that he has his MasterCard and the attendant doesn't have a coronary. If Alien Bob gets gas the attack on Washington will succeed if the anti-gravity device doesn't fail. So if Alien Bob has his MasterCard and the attendant doesn't have a coronary, the attack on Washington will succeed unless the anti-gravity device fails.

Instructions: For each of the arguments below construct a translation key and translate the argument into symbolic notation.

1. If Paula goes to the beach, she won't go to the movies. If she goes to the movies only if she doesn't go to the beach, Ronald will take her shopping if and only if she gets home early. But she won't get home early. So Ronald won't take her shopping.

2. Unless Fred joins the club, neither Harry nor Bill will join. But if Harry doesn't join, Paula will become the new president only if Jake resigns. However, Jake won't resign. So, if Fred doesn't join the club, Paula won't become the new president.

3. Sally will go to dinner with Bob just in case he gets his BMW out of the shop but doesn't charge the limit on his Visa card. If Bob doesn't get his BMW out of the shop, he won't charge the limit on his Visa card; and he will charge the limit on his Visa card if he does get the BMW out of the shop. Provided that Sally doesn't go out to dinner with Bob, she'll go out with Oscar; however, if she goes out with Oscar, Oscar will charge the limit on his MasterCard. So if Bob doesn't get the BMW out of the shop, Oscar will charge the limit on his MasterCard.

4. Bill made it to class on time provided that his car was working and he didn't stay up too late the night before. But if he made it to class on time, then unless he fell asleep he learned about Freud's life. It isn't true that he either fell asleep or didn't stay up too late the night before. So, Bill didn't learn about Freud's life only if his car wasn't working.

5. In spite of the fact that Lynn is a hard worker, her boss doesn't pay her well. But she can't get ahead unless she's both a hard worker and her boss pays her well. And she can't buy the new car she wants if she can't get ahead. So she's not going to be able to buy the new car she wants.

6. Dracula ate well only if the tourists visited the castle and stayed the night. However, the tourists visited the castle and stayed the night if and only if they were either idiots or their car broke down. Therefore, since the tourists were not idiots, Dracula did not eat well unless their car broke down.

7. The Reverend Mantis had time to start praying only if Mrs. Mantis didn't have dinner after sex. However, unless Mrs. Mantis had dinner after sex, neither the black Widow nor Lady Bug had dessert. Therefore, since both Lady Bug and the black Widow had dessert, the Reverend Mantis didn't have time to start praying.