CHAPTER 8
INDUCTIVE LOGIC

In this chapter we are going to be examining some arguments whose conclusions do not follow with certainty from their premises, but are only more or less likely, given those premises. Although these arguments cannot provide us with certainty, we would at least like them to be able to give us a degree of confidence in the claim they are reasoning about; and we would like to know how to strengthen these arguments, to the extent that it is possible for us to do so.

As we noted earlier, such arguments are inductive, rather than deductive. So we are going to be briefly exploring the realm of inductive logic.

There are two types of inductive arguments that seem especially worth examining, if only because we use them so often in life: Arguments by Analogy, and Causal Arguments. Let's briefly consider each of these, and then close the chapter by discussing classical probability theory.

ARGUMENTS BY ANALOGY

When we start running low on toothpaste many of us go to the grocery store and select the same brand we have often purchased in the past. Why? Perhaps it's mostly a matter of habit. However, there is a rational basis to our selection, and if we were to formulate it in terms of an argument, we might say something like: "I have purchased this brand frequently in the past and have always found it effective in preventing cavities. So I will probably find this package of toothpaste effective too."

Notice that the conclusion of this argument is only likely. We could get the toothpaste home, start using it, and find that it actually encourages cavities. Unfortunately, that is exactly the problem with inductive arguments. They can't provide us with certainty. Sometimes, though, they're all we've got.

Let's develop a little terminology, and explore our toothpaste example a bit more. In these cases, we are always reasoning about some object; and we are trying to establish that it has some characteristic, or property, like being effective in fighting cavities. We are also referring back to similar objects that we know had that property. Let's call the object we are trying to conclude something about "the object we're reasoning about," and the property we are reasoning that this object has "the key property." Further, let's call the objects we are referring back to "the objects in the comparison class," and the respects in which we know that these objects resemble the one we're reasoning about "the resembling properties."

Clearly, the conclusion of our argument is that the object we're reasoning about has the key property. Why? Because the objects in the comparison class had it, and they resemble it in many other respects (i.e., there are many resembling properties between the two.)

Armed with these terms we can now make several points, most of which are obvious yet are often overlooked, about these sorts of arguments.

First, the more objects there are in the comparison class the stronger our argument is likely to be. Surely, if we have had experience with only a few packages of a particular brand of toothpaste our conclusion is less likely to be true than it is if we have had experience with many such packages.

Oddly enough, people frequently violate this principle and reason based on far too few objects in the comparison class. Indeed, sometimes they seem to think all that is required is one such object. (This is especially true when it comes to buying expensive items like cars. We have often heard people reason that they are going to buy a certain car because someone they know bought one and never had any trouble with theirs.)

Second, the more resembling properties there are the stronger our argument is likely to be. Ideally, we would like the objects in the comparison class to resemble the object we're reasoning about in every respect. Yet this is surely not possible since, for example, the old packages of toothpaste were bought before the one we are now considering buying, and they may even have been manufactured in a different factory, etc. Still, we would like the object we are reasoning about to strongly resemble them.

Apparently, consumers frequently overlook this point too. When they see "new and improved" on the label, they evidently reason that if the toothpaste was good before, it will be even better now. They fail to notice that the argument is actually weaker, precisely because there are fewer resembling properties.

Third, for our argument to be at all effective, the resembling properties should have something to do with the production of the key property. That is why, in the toothpaste case, for instance, we should be more concerned with the contents of the package than we are with the packaging. Of course here too, the consumer is often bamboozled.

Fourth, the weaker the conclusion of the argument is, relative to its premises, the stronger it is likely to be. Suppose we have used Frosty Fresh toothpaste the past ten years and have never had a cavity in that time. If we conclude based on this evidence that probably we won't have any cavities while this package of Frosty Fresh lasts, our argument will be much weaker than if we conclude that we probably won't have many cavities while we use this package of Frosty Fresh.

Our final point concerns a case where although we know that our new object has one of several properties we don't know exactly which of these it has. (Suppose, for example, we know Frosty Fresh is manufactured in four different locations. We might not know which of these locations the package on the shelf in front of us was manufactured in, but we do know that it was manufactured in one of the four.) In these kinds of cases we want the objects in the comparison class to vary among themselves, within that range of properties, as much as possible. The reason for this is simple. What we don't want is for the object we are reasoning about to have a property that is different from the objects in the comparison class and is causally significant in the production of the key property.

These are a few of the more important points that need to be born in mind when we are dealing with Arguments by Analogy. Let's try some questions.

PROBLEMS

How about horses? Does only one man harm them, while others do them good? Is not the exact opposite the truth? One man is able to do them good, or at least not many. Is it not true that the trainer of horses does them good, and others who have to do with them rather injure them? Is not that true, Meletus, of horses, or of any other animals? Most assuredly it is; whether you and Anytus say "yes" or "no". Happy indeed would be the condition of youth if they had one corrupter only, and all, the rest of the world, were their improvers. (Plato's Apology.)

1. Answer a, b, c, d, or e.
   What is the object being reasoned about in the passage above?
   a. The horses
   b. The youth
   c. The trainer
   d. The corrupter
   e. The one who does them good?
2. Answer a, b, c, d, or e.
   What are the objects in the comparison class?
   a. The horses
   b. The youth
   c. The trainer
   d. The corrupter
   e. The one
3. Answer s, w, or n.
   Is the Analogical argument in this passage stronger, or weaker, or neither stronger nor weaker, than our toothpaste example above?
4. Answer a, b, c, d, or e.
   Why did you answer question 3 in the way you did?
   a. There aren't enough objects in the comparison class.
   b. There aren't enough resembling properties.
   c. The resembling properties have little to do with the production of the key property.
   d. The conclusion is too strong relative to the premises.
   e. The objects in the comparison class don't vary among themselves enough.

5. Answer a, b, c, d, or e.

The biggest problem with the Analogy above is that:

a. There aren't enough objects in the comparison class.
b. There aren't enough resembling properties.
c. The resembling properties have little to do with the production of the key property.
d. The conclusion is too strong relative to the premises.
e. The objects in the comparison class don't vary among themselves enough.

CAUSAL ARGUMENTS

Suppose you are a health inspector. Several people have recently come down with ptomaine poisoning. Your job is to find out what is causing it and, if possible, correct the problem.

Clearly what you need here is an argument whose conclusion is of the form, "A caused P," where P is the phenomenon you are trying to causally explain, namely, ptomaine poisoning, while A is its cause. What are missing are not only the premises, but also knowledge of precisely what A is. How are you going to find this out?

You begin your investigations by talking with those people who have recently gotten ptomaine poisoning. You discover that they all came down with the illness during the sweltering last two weeks of August, and that they had all been swimming shortly before they became ill. Many of them had also shopped at the Big Chain Department Store, seen the new movie "Violent Affair" at the Strand Cinema, and watched television. You remember from the logic class you took in college many years ago that you cannot reason, for example, that the sweltering heat caused the illness. To do so would be to commit the False Cause Fallacy. How should you go about reasoning to discover the cause of the illness?

In the 19th Century, John Stuart Mill proposed five methods he claimed could be used to provide inductive evidence to support a claim of the sort, "A caused P." Where "P" is the phenomenon we want to causally explain (viz., ptomaine poisoning) and A, B, . . ., O, are antecedent circumstances (i.e., the events that occurred before the phenomenon we are trying to explain: the sweltering heat and the fact that those people who got ptomaine poisoning had gone swimming shortly before they became ill, etc.), these methods can be summarized as follows:

1. THE METHOD OF AGREEMENT

To use this method we need several different cases where the phenomenon in question did occur. Suppose you have questioned five people who came down with the illness. After a more thorough examination you discover that, while many antecedent circumstances differed from case to case, there were, in fact, four such circumstances shared by all. Not only had they become ill when it was hot (which we will represent by the letter "H"), and gone swimming (S), but they had also eaten at the Greasy Spoon (G), and had stopped by for dessert at a place called "Calorie Heaven" (C). Mill's Method of Agreement, in effect, now tells us that we have some justification for asserting that the ptomaine poisoning was probably caused by one of these four antecedent circumstances. Schematically, we can formulate this argument as follows:

CASE	ANTECEDENT CIRCUMSTANCES								PHENOMENON
1.	A,	-,	C,	G,	H,	S,	V,	-,	X	P
2.	A,	B,	C,	G,	H,	S,	V,	W,	-	P
3.	A,	B,	C,	G,	H,	S,	-,	W,	X	P
4.	-,	B,	C,	G,	H,	S,	V,	W,	X	P
5.	A,	B,	C,	G,	H,	S,	V,	W,	X	P

Probably C, G, H, or S caused P.

2. THE METHOD OF DIFFERENCE

To use this method, we need only two cases. But they must be extremely similar. Suppose then that you discover that one of the people who came down with ptomaine poisoning had a brother with whom he spent the entire weekend. Oddly enough, his brother did not become ill. On questioning him you discover that the only differences between the two were that the brother that didn't become ill, didn't go shopping at the Big Chain Department Store, didn't go swimming, and didn't eat at the Greasy Spoon. Schematically, you set the case up as follows:

CASE	ANTECEDENT CIRCUMSTANCES									PHENOMENON
1.	A,	B,	C,	G,	H,	S,	V,	W,	X	P
2.	A,	-,	C,	-,	H,	-,	V,	W,	X	-

Probably, B, G, or S caused P.

3. THE JOINT METHOD OF AGREEMENT AND DIFFERENCE

This method is really just a combination of the two methods we have already considered. Suppose we include the brother who did not get ptomaine poisoning in the list of people. We can then conclude that probably either eating at the Greasy Spoon, or going swimming, is the cause of the illness. Schematically, the case can be formulated as follows:

CASE	ANTECEDENT CIRCUMSTANCES									PHENOMENON
1.	A,	-,	C,	G,	H,	S,	V,	-,	X	P
2.	A,	B,	C,	G,	H,	S,	V,	W,	-	P
3.	A,	B,	C,	G,	H,	S,	-,	W,	X	P
4.	A,	B,	C,	G,	H,	S,	V,	W,	X	P
5.	A,	B,	C,	G,	H,	S,	V,	W,	X	P
6.	A,	-,	C,	-,	H,	-,	V,	W,	X	-

Probably, G or S caused P.

4. THE METHOD OF RESIDUES

This method tells us that if, from a list of antecedent circumstances and subsequent phenomena, we know the causal impact of all of these antecedent circumstances except one, the antecedent circumstance whose causal impact we do not know is probably the cause of that phenomenon whose cause we do not know. Thus, suppose we know that someone who shopped at the Big Chain, ate at the Greasy Spoon, and went swimming, became depressed, got sunburned, and came down with ptomaine poisoning. Suppose we also know that shopping at the Big Chain Department Store causes depression, and swimming causes sunburns. Then we have some reason for thinking that eating at the Greasy Spoon causes ptomaine poisoning. Schematically, we can represent this as follows:

CASE	ANTECEDENT CIRCUMSTANCE		PHENOMENA
1.	B, S, G		D, R, P
	B	caused	D.
	S	caused	R.

Probably G caused P.

5. THE METHOD OF CONCOMITANT VARIATION

This method deals with cases, which involve a variation in intensity not only of antecedent circumstances, but also of phenomena. Suppose, in the example we have been discussing, we have discovered three sisters, all of whom shopped at the Big Chain, ate at the Greasy Spoon, and went swimming. Suppose further, that they all became depressed, got sunburned, and suffered from a greater or lesser degree of ptomaine poisoning. In addition to this, suppose the sister who ate the most got the worst case of ptomaine poisoning, while the sister who ate the least got the mildest case of ptomaine poisoning. In this case we have some further evidence that eating at the Greasy Spoon causes ptomaine poisoning. Schematically we can represent this as follows:

CASE	ANTECEDENT CIRCUMSTANCES	PHENOMENA
1.	B, S, G+	D, R, P+
2.	B, S, G	D, R, P
3.	B, S, G-	D, R, P-

Probably G caused P.

While these methods are not foolproof, and can at most provide us with some justification for a claim of the form, "A caused P," people who are trying to determine the cause of a given phenomenon still frequently use them.

PROBLEMS

Instructions: Determine which Method is being used in each of the passages below? Answer a, b, c, d, or e.

a. The Method of Agreement
b. The Method of Difference
c. The Joint Method of Agreement and Difference
d. The Method of Residues
e. The Method of Concomitant Variation

1. Jack: You're going to have to cut down on your smoking Bill, because the more smoking you do, the worse your cough gets.

2. Since the only difference between the way in which you cared for your peach trees this year and last, is that you didn't use dormant spray on them this year, but you did last year, and since they have developed peach leaf curl this year, while they didn't last year, your failure to spray them probably is the cause of the problem.

3. All of those students who turned their homework assignments in on time got A's or B's on the Examination. So probably they did well on the Exam because they did their homework and got it in on time.

4. If you don't think that a helmet works, try the following experiment: Put a helmet on, and ask someone to hit you over the head with a baseball bat. Now take the helmet off. Have him hit you over the head again. If you can't tell the difference, you're probably too hardheaded to need a helmet.

5. Harry: How do you know that a faulty fuel pump was causing the car to stall?
Larry: On the bill it said they had replaced the fan belt, the starter, and the fuel pump. But I knew the car was overheating because the fan belt was loose; and I knew that I frequently couldn't get the engine to turn over in the morning because the starter was broken. So I guess the faulty fuel pump must have been causing the car to stall.

6. Betty Noire: Whenever I hit my brother, and kick him in the stomach, he grabs his belly and runs screaming to mom. Moreover, the harder I hit him the louder he screams. If I don't hit him he stops screaming, and if I just kick him in the stomach he runs to mom, but he doesn't scream.

Sera Phim: Maybe if you stopped hitting your brother he wouldn't be such a crybaby.

CLASSICAL PROBABILITY THEORY

There are several senses in which we use the term 'probable.' We may, for example, say that it is probable that the U.S. will lose the war in Afghanistan, or that it is probable that a person will not live past his or her 100^{th} Birthday, or that it is probable that the next card drawn from a deck of cards will not be a face card. While our knowledge that it is probable that a person will not live past his or her 100^{th} birthday is obtained by taking a large sample of people and determining what percentage of them live beyond their 100^{th} birthday, our knowledge that it is probable that the next card drawn from a deck of cards will not be a face card is more likely to be obtained in an *a priori* manner. In this last case the assumptions are made that we know how many cards are involved, that it is equally probable for any card in the deck to be selected as any other, and, since there are fewer face cards in the deck than non-face cards, that the likelihood is that a non-face card will be selected. This *a priori* theory of probability is often called Classical Probability.

According to Classical Probability Theory, the probability of an event a occurring, represented as P(a), equals the number of favorable outcomes divided by the number of possible outcomes. So if we are dealing with a standard deck of 52 playing cards the probability of drawing an ace equals the number of aces in the deck, viz. 4, divided by the total number of cards in the deck, viz. 52. Also, it is assumed that for any event a, $0 \leq P(a) \leq 1$, and if a is impossible then P(a) = 0, while if a is certain then P(a) = 1. Finally, equivalent statements are assumed to have equivalent probabilities

Multiple events are then handled in terms of the following principles:

1. The Probability of Joint Occurrences: What is the probability of both of two events, a and b, occurring:

a. Where one of the events is *dependent* on the other (i.e., where the occurrence of a has an effect on the occurrence of b)?

The General Multiplicative Law: P(a & b) = P(a) x P(b given a).

Example: What is the probability of getting two aces on two successive draws from a standard 52 card deck when the first card is not replaced before the second card is drawn?

Solution: P(a) = 4/52 and P(b given a) = 3/51. So P(a & b) = (4/52 x 3/51) = 12/2652 = 1/221.

b. Where the two events are *independent* (i.e., where the occurrence of a has no effect on the occurrence of b, and vice versa)?

The Restricted Multiplicative Law: P(a & b) = P(a) x P(b).

Example: What is the probability of getting two aces on two successive draws from a 52 card deck when the first card is replaced before the second card is drawn?

Solution: P(a) = 1/13, P(b) = 1/13. So P(a & b) = (1/13 x 1/13) = 1/169.

Obviously the Restrictive Multiplicative Law follows from the General Multiplicative Law in cases where P(b given a) = P(b).

2. The Probability of Alternate Occurrences: What is the probability of either of two events, a or b occurring:

a. In cases where both of the two events can occur (i.e., where the two events are *not mutually exclusive*)?

The General Additive Law: P(a or b) = P(a) + P(b) - P(a & b).

Example 1: What is the probability of getting an ace or a spade from a 52 card deck?

Solution: P(a or b) = 1/13 + 1/4 - (1/52).
= 4/52 + 13/52 - 1/52
= 16/52
= 4/13

Example 2: What is the probability of getting an ace from a 52 card deck on two draws from the deck, when the first card is replaced before the second card is drawn?

Solution: P(a or b) = 1/13 + 1/13 - (1/13 x 1/13)
= 13/169 + 13/169 - (1/169)
= 25/169

b. In cases where the two events are *mutually exclusive* (i.e., exactly one of the two events can occur but not both)?

The Restricted Additive Law: P(a or b) = P(a) + P(b).

Example: What is the probability of getting either an ace or a king from a 52 card deck?

Solution: P(a or b) = 1/13 + 1/13 = 2/13.

Obviously the Restrictive Additive Law follows from the General Additive Law in cases where P(a & b) = 0.

3. Rather than attempting to directly determine the probability of a given event it is sometimes easier to determine what the probability of that event not occurring is, and then use the following rule to find the answer:

The Negation Law: P(a) = 1 – P(ā).

This rule relies on the fact that where ā represents the event of a not occurring, P(a) + P(ā) = 1. Using this rule in the example above we get:

P(a) = 1 – (12/13 x 12/13)
= 1 – 144/169
= 25/169.

Example: What is the probability of drawing at least one white ball on two successive draws from an urn containing two red balls and six white ones, when the first ball is not returned before the second one is drawn?

Solution: In this case the problem becomes easier if we ask ourselves what the probability of not getting a white ball on either draw is. Clearly we would need to draw two red balls, and the probability of that occurring is 1/4 x 1/7 = 1/28. So the probability of getting at least one white ball is 1 – 1/28 = 27/28.

4. Since P(a & b) = P(b & a), by the General Multiplicative Law we obtain:

P(a given b) x P(b) = P(b given a) x P(a).

Dividing both sides by P(b), this becomes:

P(a given b) = [P(b given a) x P(a)] / P(b).

This formula is sometimes useful.

Example: Suppose that 48% of all households have a dog and 9.6% of all households have both a dog and a cat. What is the probability that a household has a cat given that they have a dog?

Solution: Here P(d) = .48 and P(c & d) = .096. So P(c given d) = .096 / .48 = 20%.

5. From the above formula, since P(b) = P(b & a) v P(b & ā), substituting P(b given a) x P(a) for P(b & a) and P(b given ā) x P(ā) for P(b & ā) we have:

Bayes' Theorem: [P(a given b) = [P(b given a) x P(a)] / {[P(b given a) x P(a)] + [P(b given ā) x P(ā)]}.

Example: A class of students contains 70% Asians. 10% of the Asians speak French while 50% of the non-Asians speak French. If a student is chosen at random and found to speak French, what is the probability that he or she is Asian?

Solution: Given P(a) = 70%, P(ā) = 30%, P(f given a) = 10%, P(f given ā) = 50%. What is P(a given f)?

P(a given f) = [P(f given a) x P(a)] / {[P(f given a) x P(a)] + [P(f given ā) x P(ā)]}

= (.1 x .7) / [(.1 x .7) + (.5 x .3)]
= .07 / .07 + .15
= .07 / .22
= .318

*The Monty Hall Problem***:** In a T.V. game show entitled "The Monty Hall Show," a contestant was asked to select one of three doors, only one of which contained a significant prize. After the contestant selected a door, Monty Hall would open one of the other doors that did not contain the prize. He would then ask the contestant if he or she wanted to select another door. The problem is: if you were the contestant, what should you do?

Solution: Suppose you select door a, and when Monty Hall opens door b you stick with door a.

P(a given Monty opens b) = P(a) x P(Monty opens b given a) / P(Monty opens b) = (1/3 x 1/2) / (1/2) = 1/3. So you have a 1/3 chance of selecting the correct door.

But now suppose you switch to door c after Monty Hall opens door b.

P(c given Monty opens b) = P(c) x P(Monty opens b given c) / P(Monty opens b) = (1/3 x 1) / (1/2) = 2/3. So you double your chances by switching doors.

If you are still not convinced, maybe the following explanation will help. The probability that you chose the correct door initially is 1/3, since there are three doors, each of which has an equal chance of concealing the prize. The probability that the door Monty Hall chooses conceals the prize is 0, since he *never* chooses the door that contains the prize. Since the sum of the three probabilities is 1, the probability that the prize is behind the other door is 1 - (1/3 + 0), which equals 2/3. Therefore you will double your chance of winning by switching doors.

EXPECTED VALUE

Suppose someone offers to bet you $2 that you can't throw a 7 on a single roll of a fair pair of dice. You quickly calculate that there are only six ways of winning (viz. 6-1, 1-6, 5-2, 2-5, 4-3, and 3-4) and thirty ways of losing (viz. 6-2, 2-6, 6-3, 3-6, 6-4, 4-6, 6-5, 5-6, 6-6, 5-1, 1-5, 5-3, 3-5, 5-4, 4-5, 5-5, 4-1, 1-4, 4-2, 2-4, 4-4, 3-1, 1-3, 3-2, 2-3, 3-3, 2-2, 2-1, 1-2, 1-1), so you explain that you only have a 1in 6 chance of winning, and you refuse the offer. The bet is now altered. You put up $2 and will be given $10 if you can throw a 7. Should you take the bet? This is where the concept of expected value becomes relevant. To obtain the expected value of a given outcome multiply the probability of each outcome times the amount gained or lost by selecting that outcome and divide the sum of these by the total number of possible outcomes. In the case we are considering there are 6 chances of getting a 7 and each of these will pay you $10, minus the $2 you had to pay to place the bet, so 6 x $8 = $48. On the other hand, there are 30 cases in which you will lose the $2, so that is 30 x -$2 = -$60. Adding the two ($48 and -$60) and dividing by the total number of possible outcomes we get an expected value of -12/30 = -$.40. Based on this reasoning you again refuse the bet.

PROBLEMS

1. When a pair of dice is rolled twice, what is the probability of getting a 7 on both rolls?

2. When a pair of dice is rolled, what is the probability of getting either a 7 or an 11?

3. A die is rolled five times. What is the probability of getting two twos?

4. Three crazy Cossack officers are playing Russian roulette using one bullet and a six chambered pistol. The officer who is to go third gets to decide whether or not the cylinder of the gun should be spun after each pull of the trigger. Assuming he wants to live, should he select the spin-the-cylinder version, and what are the probabilities that he will survive if he selects this version?

Suppose a new round is started, but this time the person who goes third has to choose whether to select a gun with only five chambers and be permitted to select whether or not to play the spin-the-cylinder version, or select a gun with six chambers but where no cylinder spinning is allowed. Assuming this person wants to live what should he do?