CHAPTER 9
CATEGORICAL SYLLOGISMS

STANDARD-FORM CATEGORICAL STATEMENTS

In this chapter we are going to explore a special kind of argument known as a categorical syllogism. A categorical syllogism is a deductive argument that has exactly two premises and contains only categorical statements. A categorical statement is a statement that asserts that either a part of, or the whole of, one set of objects -- the set identified by the subject term in the sentence expressing that statement -- either is included in, or is excluded from, another set -- the set identified by the predicate term in that sentence.

For a categorical statement to be in standard-form, the sentence expressing that statement must begin with the quantifier "all," "no," or "some." It must then present the subject term -- the term designating the set of objects the statement is about -- followed by the copula -- either "are" or "are not" -- followed, finally, by the predicate term. So, for a categorical statement to be in standard-form, the sentence that expresses it must have precisely the following structure:

Quantifier + Subject Term + Copula + Predicate Term

There are exactly four standard-form categorical statements, each of which is identified with a capitalized vowel of the alphabet. They are:

A: All S are P.	Example: All birds are mammals.
E: No S are P.	Example: No birds are reptiles.
I: Some S are P.	Example: Some birds are sparrows.
O: Some S are not P.	Example: Some birds are not carnivores.

1. THE QUANTITY AND QUALITY OF STANDARD-FORM STATEMENTS

The words “all,” “no,” and “some” are called “quantifiers” because they specify a quantity. “All” and “no” are universal quantifiers because they refer to every object in a certain set; while the quantifier “some” is a particular, or existential, quantifier because it refers to at least one existing object in a certain set.

A categorical statement is said to have a universal quantity when the sentence that expresses it begins with a universal quantifier. It is said to have a particular quantity, on the other hand, when the sentence that expresses it begins with a particular quantifier. Thus, both A- and E-statements have a universal quantity, while both I- and O-statements have a particular quantity.

Besides having either a universal or particular quantity, standard-form categorical statements also have either affirmative or negative quality. The statements, all birds are mammals, and some birds are sparrows, have affirmative quality because they assert something about the inclusion of the set of birds in the set of mammals or sparrows. While the statements, no birds are reptiles, and some birds are not carnivores, have negative quality because they deny that any members of the set of birds are included in the set of reptiles in the former case, and they deny that some members of the set of birds are included in the set of carnivores in the latter case.

2. THE DISTRIBUTION OF TERMS IN SENTENCES EXPRESSING CATEGORICAL STATEMENTS

Another important pair of concepts concerns both the subject and predicate terms of sentences that express categorical statements. A term is distributed in such a sentence if it refers to all members of the set of objects denoted by that term. Otherwise, it is said to be undistributed.

In a sentence expressing an A-statement, e.g., "All birds are mammals," the subject term "birds" is distributed, while the predicate term "mammals" is undistributed. On the other hand, in a sentence expressing an E-statement, e.g., "No birds are reptiles," both the subject and predicate terms are distributed. In a sentence expressing an I-statement, e.g., "Some birds are sparrows," neither the subject nor the predicate term is distributed. Finally, in a sentence expressing an O-statement, e.g., "Some birds are not carnivores," only the predicate term is distributed.

We can summarize these points as follows:

STATEMENT-FORM	*QUANTITY*	*QUALITY*	*DISTRIBUTED*
A: All S are P.	Universal	Affirmative	S only
E: No S are P.	Universal	Negative	Both S and P
I: Some S is P.	Particular	Affirmative	Neither S nor P
O: Some S is not P.	Particular	Negative	P only

MANIPULATING SENTENCES

In the next section we are going to be developing some concepts that involve manipulating sentences in various ways. The present section helps prepare the way for this.

Suppose, for whatever reason, we decide we want to change the quantity of a statement without altering its quality. If the sentence which expresses this statement begins with the universal quantifier "all," the only thing we need to do is replace that word with the particular quantifier "some." For example, we just replace, "All birds are mammals," with "Some birds are mammals." However, suppose the sentence begins with the universal quantifier "no." Suppose, for example, it reads, "No birds are reptiles." If we simply replace the word "no" with "some," and thus obtain, "Some birds are reptiles," we have not only altered the quantity, but the quality as well. For, although the statement this sentence expresses does have a particular quantity, it also has an affirmative quality, whereas the original statement had a negative quality. What we must do instead, then, is to write, "Some birds are not reptiles."

An important concept we need to be familiar with is the complement of a set. The complement of a set, A, is the set of all those things that are not As. Thus, the complement of the set of birds is the set of things that are not birds. To express this set in a sentence we will replace the term "birds" with the term "non-birds." For example, when we are instructed to replace the predicate term, in the sentence, "Some birds are sparrows," with its complement, we will write, "Some birds are non-sparrows."

1. CONVERSION, OBVERSION, AND CONTRAPOSITION

One categorical statement, S1, is the converse of another, S2, if and only if the sentence expressing S1 is the result of switching the subject and predicate terms in the sentence expressing S2. So, for example, the statement that all mammals are birds is the converse of the statement that all birds are mammals; while the statement that some carnivores are not birds is the converse of the statement that some birds are not carnivores.

Obversion is a more complicated operation than conversion. To obtain the obverse of a categorical statement we must first replace the predicate term in the sentence expressing that statement with its complement, and then change the quality. So the obverse of the statement that all birds are mammals is the statement that no birds are non-mammals; while the obverse of the statement that some birds are sparrows is the statement that some birds are not non-sparrows.

Finally, contraposition is even a more involved operation than obversion. To obtain the contrapositive of a statement, we first replace the subject and predicate terms in the sentence that expresses this statement and then exchange both these terms with their complements. So, the contrapositive of the statement, "All birds are mammals," is the statement, "All non-mammals are non-birds"; while the contrapositive of the statement, some birds are mammals, is the statement, some non-mammals are non-birds.

2. CONTRADICTORY, CONTRARY, SUBCONTRARY, AND SUBALTERNATION

Typically the notions we are going to begin discussing in this section are developed in terms of truth-values. For the moment, however, it might prove useful to introduce them in a different way.

Two statements are contradictory if and only if the sentences expressing them have the same subjects and predicates, but they differ in both quantity and quality. Thus, the statements that all birds are mammals and that some birds are not mammals, are contradictory; and so also are the statements that no birds are reptiles and that some birds are reptiles.

Two statements are contrary if the sentences expressing them have the same subjects and predicates, and both begin with universal quantifiers, but they differ in quality. Thus, the statements, all birds are mammals, and no birds are mammals, are contraries.

Two statements are subcontraries if and only if the sentences expressing them have the same subject and predicate terms, and both begin with existential quantifiers, but they differ in quality. So, for example, the statements that some birds are sparrows and that some birds are not sparrows, are subcontraries.

Finally, the relation of subalternation will be said to obtain between two statements when the only respect in which they differ is their quantity. Here the statement that has a universal quantity is called the "superaltern," while the statement that has a particular quantity is called the "subaltern." Thus, the relation of subalternation obtains between the statement that all birds are mammals and the statement that some birds are mammals; and in this relation, the statement, all birds are mammals, is the superaltern, while the statement, some birds are mammals, is the subaltern. Similarly, the relation of subalternation obtains between the statement that no birds are reptiles and the statement that some birds are not reptiles; and in this relation the superaltern is the statement that no birds are reptiles, while the subaltern is the statement that some birds are not reptiles.

MULTIPLE-CHOICE QUESTIONS

Instructions: Answer a, b, c, d, e, f, or g.

a. All savings and loans are insolvent institutions.
b. No savings and loans are insolvent institutions.
c. Some savings and loans are not insolvent institutions.
d. Some insolvent institutions are savings and loans.
e. No savings and loans are non-insolvent institutions.
f. All savings and loans are non-insolvent institutions.
g. Some non-insolvent institutions are non-savings and loans.

1. The statement that results from changing both the quantity and quality of the statement that some savings and loans are insolvent institutions is:
2. The statement that results from changing only the quality of the statement that some savings and loans are insolvent institutions is:
3. The contrapositive of the statement that some savings and loans are insolvent institutions is:
4. The superaltern of the statement that some savings and loans are insolvent institutions is:
5. The subcontrary of the statement that some savings and loans are insolvent institutions is:
6. The converse of the statement that some savings and loans are insolvent institutions is:

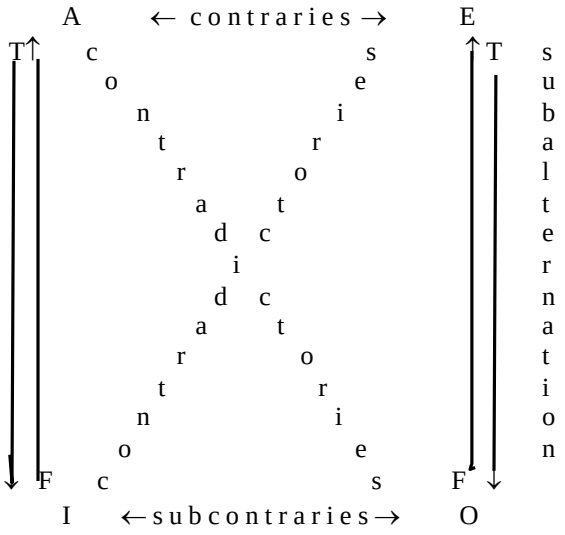
THE SQUARE OF OPPOSITION AND IMMEDIATE INFERENTIAL RELATIONS IN CLASSICAL LOGIC

So far all we have been doing is introducing terminology that deals with manipulating certain sentences to obtain new statements from ones originally given. We are now ready to begin discussing some principles of reasoning that were believed to hold in classical logic. (Aristotle invented classical logic over 2,000 years ago.) We will begin with some principles that involve inferences from one statement to another.

The classical view of contradiction maintained that two statements were contradictory just in case they had to have different truth-values. Thus, A- and O-statements were contradictories, and so also were E- and I-statements. Two statements were said to be contraries if and only if at least one of them was false, and they could not be both true. Thus, A- and E-statements were said to be contraries. Two statements were said to be subcontraries just in case at least one was true and they could not be both false. Thus, both I- and O-statements were held to be subcontraries. Finally, one statement was said to be the superaltern of another if and only if its truth entailed the truth of the other, while the other's falsity entailed its falsity. Thus, A-statements were held to be the superalterns of I-statements, and E-statements were held to be the superalterns of O-statements.

These principles were illustrated graphically by means of the Square of Opposition.

THE SQUARE OF OPPOSITION



A: All S are P.
E: No S are P.
I: Some S are P.
O: Some S are not P.

To use this diagram, suppose we know the claim that all whales are mammals is true. Since this is an A-statement, we know that the contradictory, some whales are not mammals, is false; and since this O-statement is false, and it is the subaltern of the E-statement that no whales are mammals, this latter statement is also false. Finally, since this E-statement is false, its contradictory is true. Therefore, the statement that some whales are mammals is true. So, by using the Square of Opposition we have identified the truth-values of all four of the standard-form categorical statements.

Unfortunately the Square of Opposition does not always provide us with truth-values for all of the standard-form statements. Suppose, for example, the A-statement had been false instead of true. Suppose it had been that all whales are fish. The Square of Opposition now tells us that the contradictory O-statement, some whales are not fish, is true. Unfortunately, that is all the information the Square will give us here. In this instance the other two statements are therefore said to have undetermined truth-values.

QUESTIONS

Now let's see if you can use the Square of Opposition to answer the following questions.

1. Which of the four forms is the statement that some magicians are illusionists (answer A, E, I, or O)?
2. If the statement that some magicians are illusionists is true, can you determine the truth-value of the corresponding E-statement (answer Y or N)?
3. If you answered Y to question 2, then is the corresponding E-statement true or is it false (answer T or F)?
4. Can you determine the truth-value of the corresponding O-statement (answer Y or N)?
5. If you answered Y to question 4, then is the corresponding O-statement true, or is it false (answer T or F)?

THE PRINCIPLES OF CONVERSION, OBVERSION, AND CONTRAPOSITION

If we add the following principles concerning conversion, obversion, and contraposition, to the above account, the Square of Opposition can help us obtain answers to many other questions.

The Principle of Conversion: The converse of an E- or an I-statement is logically equivalent to the original.

Thus, for example, the statement that some birds are sparrows is logically equivalent to the statement that some sparrows are birds.

Note, however, that the above principle does not apply to either A- or O-statements. For example, the statement that all birds are mammals is not equivalent to the statement that all mammals are birds; nor is the statement, some birds are not sparrows, equivalent to the statement, some sparrows are not birds.

The Principle of Obversion: The obverse of any of the four types of statements is logically equivalent to the original.

Thus, the statements that all birds are mammals and that no birds are non-mammals are equivalent; and so also are the statements that some birds are sparrows and that some birds are not non-sparrows.

The Principle of Contraposition: The contrapositive of an A- or an O-statement is logically equivalent to the original.

For example, the statement that all birds are mammals is logically equivalent to the statement that all non-mammals are non-birds; and so also are the statements that some birds are not carnivores and that some non-carnivores are not non-birds.

Notice, however, that the principle does not hold for either E- or I-statements. Thus, for example, the statement that no birds are reptiles is not equivalent to the statement that no non-reptiles are non-birds; and the statement that some birds are reptiles is not equivalent to the statement that some non-reptiles are non-birds.

To see how these principles can be used with the traditional Square of Opposition to obtain results about some immediate inferences consider, for example, the following argument:

All birds are mammals.

_____

Some non-mammals are not non-birds.

We know that the statement that all birds are mammals is an A-statement, and the Principle of Contraposition informs us that this statement is logically equivalent to the statement, "All non-mammals are non-birds." The contradictory of the statement, all non-mammals are non-birds, however, is the statement, some non-mammals are not non-birds; and the contradictory of a given statement always has the opposite truth-value of that statement. So the argument is invalid since it can have a true premise and a false conclusion.

Or, consider the following argument:

No birds are reptiles.

_____

Some non-reptiles are birds.

The Square of Opposition tells us that if the premise is true then its subaltern, viz., some birds are not reptiles, is also true. The Principle of Contraposition, however, informs us that this O-statement is equivalent to the statement, some non-reptiles are not non-birds, and this is the obverse of the statement, some non-reptiles are birds. Since the Principle of Obversion holds for all four types of categorical statements it follows that the conclusion of the argument must be true, if its premise is true. Therefore the argument is valid.

Now let's see if you can do it. Is the following a valid argument (Y/N)?

Some ships are aircraft carriers.

_____

It isn't true that no aircraft carriers are ships.

The reasoning here goes as follows: Since the statement that some ships are aircraft carriers, is an I-statement, the Principle of Conversion tells us it is equivalent to the statement that some aircraft carriers are

ships. However, if this is a true statement then its contradictory, viz., no aircraft carriers are ships, is false. However, this is simply the negation of the conclusion. So, the argument is valid.

CATEGORICAL SYLLOGISMS

So far all we have done is to investigate categorical statements and some immediate inferences that classical logic claimed could be derived from them. (An immediate inference is an argument that has only one premise.) We turn now to the classical account of categorical syllogisms.

As was mentioned earlier, a categorical syllogism is an argument that contains only categorical statements and has exactly two premises. Though the statements need not be in standard-form, we will only deal with syllogisms that contain such statements in the present discussion.

Categorical syllogisms contain exactly three terms, the major term, the minor term, and the middle term. The major term occurs as the predicate term in the conclusion of the syllogism, while the minor term occurs as the subject term in this statement. Each of these terms must also occur in one of the two premises. The premise in which the major term of the conclusion occurs is called the "major premise," while the premise in which the minor term of the conclusion occurs is referred to as the "minor premise." Besides this, the two premises must share a term, viz., the middle term.

In representing the argument, we list the major premise first, and the minor premise directly beneath it. A line is then drawn and the conclusion is listed beneath it. Thus, it is represented as follows:

Major Premise (i.e., the premise containing the major term).
Minor Premise (i.e., the premise containing the minor term).
_____
Conclusion (i.e., quantifier + minor term + copula + major term).

Together, the mood and figure identify categorical syllogisms. The mood is determined by simply listing the standard-forms of the major premise, minor premise, and conclusion (in that order). Thus, the mood of the syllogism,

Major Premise: Some birds are sparrows.
Minor Premise: All birds are mammals.
_____
Conclusion: Some mammals are sparrows.

is IAI.

The figure of the syllogism, on the other hand, is determined by the position of the middle term in the two premises. There are four figures:

FIGURE 1	FIGURE 2	FIGURE3	FIGURE 4
- M - P	- P - M	- M - P	- P - M
- S - M	- S - M	- M - S	- M - S
_____	_____	_____	_____
- S - P	- S - P	- S - P	- S - P

The argument's structure depends on its figure and mood. To represent its structure all we need to do is list both its mood and figure. We do this by simply indicating its mood first, and then a hyphen, followed by its figure. So the structure of the argument,

Major Premise: Some birds are sparrows.
Minor Premise: All birds are mammals.
_____
Conclusion: Some mammals are sparrows.

is represented as, IAI-3. All we need to do now is to decide whether this form of argument is valid.

1. DETERMINING VALIDITY

To decide whether a categorical syllogism is valid or invalid, according to the classical view, the following four rules can be used:

1. The middle term must be distributed at least once.
2. Any term distributed in the conclusion must be distributed in a premise.
3. At least one premise must be affirmative.
4. A negative conclusion requires at least one negative premise, and vices versa.

Any argument that violates one or more of these rules will be invalid. Consider, for example, the following argument:

Some naval vessels are aircraft carriers.
Some ships are naval vessels.
_____
Some ships are aircraft carriers.

This argument (and any other argument like it that has the form III-1) is invalid because the middle term (viz., "naval vessels") is not distributed in either of the premises. (Recall that a term is distributed when it refers to all objects in a set, and that in I-statements neither the subject nor predicate terms are distributed.) For this reason it is said to commit the Fallacy of the Undistributed Middle.

The argument,

All birds are mammals.
Some birds are not sparrows.
_____
Some sparrows are not mammals.

(or any other argument that has the form AOO-3) violates rule 2, because the term "mammals" is distributed in the conclusion but not in the premise. Here the argument is said to commit the Fallacy of the Illicit Major because the major premise contains the term "mammals" which is distributed in the conclusion but not the premise. (If the undistributed term had occurred in the minor premise the fallacy committed would have been called the Fallacy of the Illicit Minor.)

The following argument (which has the form EEE-1) violates rule 3, since both of its premises are negative:

No reptiles are mammals.
No birds are reptiles.
_____
No birds are mammals.

In such cases the Fallacy of Exclusive Premises is being committed.

Finally, both the arguments,

All birds are mammals.	Some birds are sparrows.
Some reptiles are not birds.	All birds are mammals.
_____	_____
Some reptiles are mammals.	Some mammals are not sparrows.

violate rule 4. The one on the left, which has the form AOI-1, has a negative premise but no negative conclusion; while the one on the right, which has the form IAO-3, has a negative conclusion but no negative premise.

We hope this provides at least a brief introduction to the central points of classical logic. We now turn to a critique of this system of logic, and an introduction to modern symbolic logic that began in the 19th century with George Boole (1815-1864).

2. CLASSICAL VS. MODERN LOGIC

On the classical account, both A- and E-statements are said to have existential import, since the corresponding I- and O-statements can validly be inferred from them. Thus, for example, from the statement, all birds are mammals, it follows that some birds are mammals; and from the statement, no birds are reptiles, it follows that some birds are not reptiles. This seems quite reasonable. But what if the subject term in the sentence that expresses an A-statement refers to objects that don't exist? What if, for example, the statement asserts that all unicorns are beautiful animals? If we evaluate this statement as true then its subaltern that some unicorns are beautiful animals must also be true. Yet this latter statement clearly commits us to the existence of unicorns. On the other hand, if we contend that the statement that all unicorns are beautiful animals is a false claim, because there are no unicorns, it follows that its contradictory, some unicorns are not beautiful animals, must be true; and this is also wrong, again, because there are no unicorns.

Because of these and other considerations, George Boole and his contemporaries were led to revise the assumptions on which classical logic was based. The original assumption that A- and E-statements had existential import was abandoned. Instead, A-statements, like all birds are mammals and all unicorns are beautiful animals, were taken as asserting that if there are any birds (or unicorns) they are mammals (or beautiful animals). While E-statements, like no birds are reptiles, were viewed as asserting that if there are any birds they are not reptiles.

Unfortunately these changes forced logicians to make several other revisions in classical logic and the traditional Square of Opposition. For example, the view that the subaltern of a true A- or E-statement was also true had to be abandoned; while the theory that contraries could not be both true also had to be rejected, since statements like, all unicorns are beautiful animals, and no unicorns are beautiful animals, were evaluated as true on the modern view, precisely because there are no unicorns. Moreover, the view that subcontraries could not both be false, proved wrong as well, since the statements that some unicorns are beautiful animals and that some unicorns are not beautiful animals, are both false, once again, because there are no unicorns.

Besides this, a fifth principle was added to the four rules for evaluating standard-form categorical syllogisms mentioned above. That rule said, "If both premises are universal the conclusion must also be universal." (Any syllogism that violated this principle was said to commit the Fallacy of Existential Import.) Accordingly, any standard-form syllogism that does not violate any of the following rules is valid on the principles of modern logic (provided, of course, that the three terms are being used univocally):

1. The middle term must be distributed at least once.
2. Any term distributed in the conclusion must be distributed in a premise.
3. At least one premise must be affirmative.
4. A negative conclusion requires at least one negative premise, and vices versa.
5. If both premises are universal, the conclusion must also be universal.

Any syllogism that violates only the fifth rule above will be valid on the classical view, but it will be invalid on the modern view.

CONCLUDING REMARKS

While it may be a bit disconcerting to study a theory and then be told it has been replaced, it is useful to keep a couple of points in mind. First, though several principles of classical logic have been abandoned or replaced, the theory works quite well unless we are talking about objects that don't exist. Moreover, it often helps us understand a revised theory better, if we know what revisions it made on the theory it supplanted, and why it made those revisions.

EXERCISES

A. IMMEDIATE INFERENCES

Instructions: Use the square of opposition, together with the principles of conversion, obversion, and contraposition, to determine whether the following immediate inferences are valid or invalid.

1. All mummies are embalmed bodies. So some embalmed bodies are not non-mummies.
2. No stoics are epicureans. So some epicureans are non-stoics.
3. Some students are procrastinators. So it is false that no procrastinators are students.
4. It is not the case that all crooks are bunco artists. So some non-bunco artists are not non-crooks.
5. Some boars are not wild animals. So it isn't true that all wild animals are boars.

B. SYLLOGISMS

Instructions: Identify the mood and figure of each argument below. Then, determine whether it is valid or invalid on both the classical and modern views. If it is invalid on either of the two theories, explain which rule or rules it violates.

1. Some arguments are valid. No sets of statements are arguments. So some sets of statements are not valid.
2. All werewolves are nocturnal beasts. No nocturnal beasts are sun-struck beachcombers. So some sun-struck beachcombers are not werewolves.
3. All bungling buglers are cacophonous clamorers. Some cacophonous clamorers are blasphemous blasters. So some blasphemous blasters are bungling buglers.
4. All amigos are friends indeed. Some friends indeed are not friends in need. So some friends in need are not amigos.
5. Some rebels are not idealists. No idealists are realists. So some realists are rebels.

Instructions: Using "liars" as the major term, "honest injuns" as the minor term, and "truth tellers" as the middle term, formulate the categorical syllogisms which have the mood and figure indicated below, and determine whether they are valid or invalid on both the classical and modern interpretations.

a. EIO-2 b. EAO-3 c. AAI-1 d. EAE-1 e. EIO-4

Instructions: Using "jungles" as the major term, "funny farms" as the minor term, and "cities" as the middle term, formulate the categorical syllogisms that have the mood and figure shown below. Then decide whether they are valid or invalid on both the classical and modern interpretations.

a. AII-3 b. OAE-4 c. AAI-1 d. IOO-2 e. AIE-3

SORITES

Arguments containing more than two premises can be handled by inferring a suppressed conclusion from two of the premises and then using the suppressed conclusion as a premise together with another premise to infer yet another conclusion, until the ultimate conclusion is finally reached. For example, the argument,

All orcas are whales.
All whales are mammals.
All mammals are warm-blooded.

_____
Some orcas are warm-blooded.

can be conceived as containing the suppressed claim that all orcas are mammals and then reformulating it as the following two separate syllogisms:

All orcas are whales.	All orcas are mammals.
All whales are mammals.	All mammals are warm-blooded.
_____	_____
All orcas are mammals.	Some orcas are warm-blooded.

The three premise argument will then be evaluated as valid on Classical Logic but invalid on Modern Logic because, although the AAA-1 argument on the left is valid, the AAI-1 argument on the right violates the rule that if both premises are universal the conclusion must be universal also.